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INTERPOLATION FUNCTIONAL POLYNOMIAL OF THE FOURTH ORDER WHICH DOES NOT USE SUBSTITUTION RULE

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Анотація. У роботі пропонується і обґрунтовується конструкція функціонального поліному типу Ньютона, яка не вимагає виконання правила підстановки, що досягається за рахунок розширення класу поліномів, у якому шукається інтерполянт.

Abstract. In this paper construction of functional polynomial of Newton type which does not use substitution rule is proposed and grounded. This is attained by means of extension of the polynomial class where interpolant is sought.

1. Introduction

The questions of generalization of classical theory of one variable function interpolation on the case of non-linear functionals and operators was investigated by series of authors. Among them there are Filipsson L., Kergin P., Ulm S., Paul V.V., Porter W.A., Prenter P.T., Sobolevsky P.I., Yanovich L.A. (see. [4–11] and the literature cited there).

In so doing, constructed interpolation formulas had integrals under signes of which were derivatives, Gateaux differentials, which can be interpolated, and Stieltjes integral by operator of scalar argument.

Authors of papers [1–3] suggested to search for interpolants of Newton type in the class of functional polynomials of the following type

$$P_n(x(\cdot)) = K_0 + \sum_{s=1}^{n} \int\limits_{0}^{1} \int\limits_{z_1}^{1} \dots \int\limits_{z_{s-1}}^{1} K_s(\overrightarrow{z}^s) \prod_{i=1}^{s} [x(z_i) - x_{i-1}(z_i)]\, dz_s \dots dz_1, \tag{1.1}$$

where as $x_i(z) \in Q[0,1]$, $i = 0, 1, \dots$ we denoted arbitrary fixed elements of the space $Q[0,1]$ – of functions semi-continuous on the segment $[0,1]$ with finite number of gap points of the first order. To find the kernel K_0, $K_s(\overrightarrow{z}^s)$, $s = \overline{1,n}$ we introduced the following continual set of knots

$x^n\left(z, \vec{\xi^n}\right) = x_0(z) + \sum_{i=1}^{n} H(z - \xi_i)\,[x_i(z) - x_{i-1}(z)]$, $z \in [0,1]$,

$\vec{\xi^n} = (\xi_1, \xi_2, \dots, \xi_n) \in \overline{\Omega}_n = \{\vec{z^n} = (z_1, z_2, \dots, z_n) : 0 \le z_1 \le z_2 \le \dots \le z_n \le 1\}$, and the following continual interpolation conditions are laid

$P_n^I\left(x^n\left(\cdot, \vec{\xi^n}\right)\right) = F\left(x^n\left(\cdot, \vec{\xi^n}\right)\right)$, $\forall \vec{\xi^n} \in \overline{\Omega}_n$, where $H(z)$ is Heaviside function.

To provide sufficient interpolationness condition of polynomial $P_n(x(\cdot))$ on continual knots $x_n\left(z, \overrightarrow{\xi}^n\right)$ one demanded the substitution rule to be satisfied. This rule is quite limitative for the functionals $F(x(\cdot))$. One can free himself of it by extending the class of polynomials (1.1). Thus in [1] it is shown that with $n = 2$ the following polynom

Key words. Interpolation polynomial, continual knot, functional polynomial, substitution rule.

$$P_2^I\left(x\left(\cdot\right)\right)=p_0^I\left(x\left(\cdot\right)\right)+p_{1,0,0}^I\left(x\left(\cdot\right)\right)+\sum_{k=0}^{1}p_{2,0,k}^I\left(x\left(\cdot\right)\right)\equiv$$
$$\equiv K_0+\int_0^1 K_{1,0,0}\left(z_1\right)\left[x\left(z_1\right)-x_0\left(z_1\right)\right]dz_1+ \tag{1.2}$$
$$+\int_0^1\int_{z_1}^1 K_{2,0,0}\left(\vec{z}^2\right)\prod_{i=1}^{2}\left[x\left(z_i\right)-x_{i-1}\left(z_i\right)\right]dz_2dz_1+$$
$$+\int_0^1 K_{2,0,1}\left(z_1\right)\prod_{i=1}^{2}\left[x\left(z_1\right)-x_{i-1}\left(z_1\right)\right]dz_1,$$

where $p_0^I(x(\cdot))=F(x_0(\cdot))$,

$$K_{p,0,0}\left(\overrightarrow{z}^p\right)=\prod_{i=1}^{p}\left[x_i\left(z_i\right)-x_{i-1}\left(z_i\right)\right]^{-1}\frac{\partial^p}{\partial z_1\ldots\partial z_p}F\left(x^p\left(\cdot,\vec{z}^p\right)\right),\quad p=1,2,$$

$$K_{2,0,1}\left(z_1\right)=\left[x_2\left(z_1\right)-x_0\left(z_1\right)\right]^{-1}\left[x_2\left(z_1\right)-x_1\left(z_1\right)\right]^{-1}\left\{\frac{d}{dz}F\left(x^2\left(\cdot,\vec{z}^2\right)\right)\Big|_{z_1=z_2}-\right.$$
$$\left.-\frac{x_2\left(z_1\right)-x_0\left(z_1\right)}{x_1\left(z_1\right)-x_0\left(z_1\right)}\;\frac{\partial F\left(x^2\left(\cdot,\vec{z}^2\right)\right)}{\partial z_1}\Big|_{z_2=z_1}\right\}$$

is interpolation on continual set of knots $x^2\left(z,\vec{\xi}^2\right)$, $0\leq\xi_1\leq\xi_2\leq 1$.

The author constructed and investigated the polynomial $P_3^I\left(x\left(\cdot\right)\right)$, which is interpolation for the functional $F\left(x\left(\cdot\right)\right)$ on continual knot $x^3\left(z,\overrightarrow{\xi}^3\right)$ $\forall\ \overrightarrow{\xi}^3\in\overline{\Omega}_3$ and has the following form

$$P_3^I(x(\cdot))=\sum_{k=0}^{3}p_k^I(x(\cdot)),$$

where $p_0^I(x(\cdot))$, $p_1^I(x(\cdot))$, $p_2^I(x(\cdot))$ are given by (1.2), and

$$p_3^I(x(\cdot))=\sum_{k=0}^{2}p_{3,0,k}^I(x(\cdot))+\sum_{k=1}^{2}p_{3,1,k}(x(\cdot))=$$
$$=\int_0^1\int_{z_1}^1\int_{z_2}^1 K_{3,0,0}\left(\overrightarrow{z}^3\right)\prod_{i=1}^{3}\left[x\left(z_i\right)-x_{i-1}\left(z_i\right)\right]dz_i+ \tag{1.3}$$
$$+\int_0^1\int_{z_1}^1 K_{3,0,1}\left(\overrightarrow{z}^2\right)\prod_{i=1}^{2}\left[x\left(z_i\right)-x_{i-1}\left(z_i\right)\right]\left[x\left(z_2\right)-x_2\left(z_2\right)\right]\ dz_2dz_1+$$

$$+\int_0^1 K_{3,0,2}(z_1)\,[x(z_1)-x_0(z_1)]\prod_{i=1}^{2}[x(z_1)-x_i(z_1)]\;dz_1+$$

$$+\int_0^1\int_{z_1}^1 K_{3,1,1}(z_1,z_3)\prod_{i=1}^{2}[x(z_1)-x_{i-1}(z_1)]\,[x(z_3)-x_2(z_3)]\;dz_3dz_1+$$

$$+\int_0^1 K_{3,1,2}(z_1)\prod_{i=1}^{3}[x(z_1)-x_{i-1}(z_1)]\;dz_1,$$

$$K_{3,0,0}\left(\vec{z}^3\right)=(-1)^3\prod_{i=1}^{3}[x_i(z_i)-x_{i-1}(z_i)]^{-1}\frac{\partial^3}{\partial z_1\partial z_2\partial z_3}F\left(x^3\left(\cdot,\vec{z}^3\right)\right)\;,$$

$$K_{3,0,1}\left(\vec{z}^{\,2}\right)=\prod_{i=1}^{2}[x_i(z_i)-x_{i-1}(z_i)]^{-1}\cdot[x_3(z_2)-x_2(z_2)]^{-1}\times$$

$$\times\left[\frac{x_2(z_2)-x_1(z_2)}{x_3(z_2)-x_1(z_2)}\cdot\frac{\partial^2F\left(x^3\left(\cdot,(z_1,z_2,z_2)^T\right)\right)}{\partial z_1\partial z_2}-\left.\frac{\partial^2F(x^3(\cdot,\vec{z}^{\,3}))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\right],$$

$$K_{3,0,2}(z_1)=[x_1(z_1)-x_0(z_1)]^{-1}\prod_{i=1}^{2}[x_3(z_1)-x_i(z_1)]^{-1}\times$$

$$\times\left[\left.\frac{\partial F(x^3(\cdot,(z_1,z_2,z_2)^T))}{\partial z_1}\right|_{z_2=z_1}-\frac{x_1(z_1)-x_0(z_1)}{x_3(z_1)-x_0(z_1)}\cdot\frac{\partial F(x^3(\cdot,(z_1,z_1,z_1)^T))}{\partial z_1}\right],$$

$$K_{3,1,1}(z_1,z_3)=\prod_{i=1}^{2}[x_i(z_1)-x_{i-1}(z_1)]^{-1}\cdot[x_3(z_3)-x_2(z_3)]^{-1}\times$$

$$\times\left[\frac{x_1(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\cdot\frac{\partial^2F\left(x^3\left(\cdot,(z_1,z_1,z_3)^T\right)\right)}{\partial z_1\partial z_3}-\left.\frac{\partial^2F(x^3(\cdot,\vec{z}^{\,3}))}{\partial z_1\partial z_3}\right|_{z_2=z_1}\right],$$

$$K_{3,1,2}(z_1)=\prod_{i=1}^{3}[x_i(z_1)-x_{i-1}(z_1)]^{-1}\times$$

$$\times\left[\frac{x_1(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\cdot\left.\frac{\partial F\left(x^3\left(\cdot,(z_1,z_1,z_3)^T\right)\right)}{\partial z_1}\right|_{z_3=z_1}-\left.\frac{\partial F(x^3(\cdot,\vec{z}^{\,3}))}{\partial z_1}\right|_{\substack{z_2=z_1\\ z_3=z_1}}\right].$$

In this paper author proposes and grounds construction of functional polynomial of Newton type $P_4^I(x(\cdot))$, which does not demand the substitution rule to be fulfulled. This is achieved by extending the class of polynomials (1.1), in which interpolant is searched.

2. Statement and solution of the problem

Let us state the problem: find such polynomial $P_4^I(x(\cdot))$, that for the functional $F: Q[0,1] \to R^1$ is interpolation on continual set of knots $x^4\left(z,\vec{\xi}^4\right)$, $0 \le \xi_1 \le \xi_2 \le \xi_3 \le \xi_n \le 1$, i. e. satisfies interpolation conditions

$P_4^I\left(x^4\left(\cdot,\vec{\xi}^4\right)\right) = F\left(x^4\left(\cdot,\vec{\xi}^4\right)\right), \quad \forall\vec{\xi}^4 \in \Omega_4.$

One searches for interpolation polynomial in the following form

$$P_4^I(x(\cdot)) = \sum_{k=0}^{4} p_k^I(x(\cdot)), \tag{2.1}$$

where $p_0^I(x(\cdot))$, $p_1^I(x(\cdot))$, $p_2^I(x(\cdot))$, $p_3^I(x(\cdot))$ are given by formulas (1.2)-(1.3),

$$p_4^I(x(\cdot)) = \sum_{k=0}^{3} p_{4,0,k}^I(x(\cdot)) + \sum_{k=1}^{10} p_{4,1,k}^I(x(\cdot));$$

$$\sum_{k=0}^{3} p_{4,0,k}^I(x(\cdot)) = \int_0^1\int_{z_1}^1\int_{z_2}^1\int_{z_3}^1 K_{4,0,0}\left(\overrightarrow{z}^4\right)\prod_{i=1}^{4}\left[x(z_i) - x_{i-1}(z_i)\right]dz_i +$$

$$+\int_0^1\int_{z_1}^1\int_{z_2}^1 K_{4,0,1}\left(\overrightarrow{z}^3\right)\prod_{i=1}^{3}\left[x(z_i) - x_{i-1}(z_i)\right]\cdot\left[x(z_3) - x_3(z_3)\right]dz_i +$$

$$+\int_0^1\int_{z_1}^1 K_{4,0,2}(z_1,z_2)\prod_{i=1}^{2}\left[x(z_1) - x_{i-1}(z_1)\right]\prod_{i=3}^{4}\left[x(z_2) - x_{i-1}(z_2)\right]\ dz_2dz_1 +$$

$$+\int_0^1 K_{4,0,3}(z_1)\left[x(z_1) - x_0(z_1)\right]\prod_{i=1}^{3}\left[x(z_1) - x_i(z_1)\right]\ dz_1;$$

$$\sum_{k=1}^{10} p_{4,1,k}^I(x(\cdot)) =$$

$$= \int_0^1\int_{z_1}^1\int_{z_2}^1 K_{4,1,1}\left(\overrightarrow{z}^3\right)\prod_{i=1}^{2}\left[x(z_1) - x_{i-1}(z_1)\right]\prod_{i=3}^{4}\left[x(z_{i-1}) - x_{i-1}(z_{i-1})\right]dz_3dz_2dz_1 +$$

$$+\int_0^1\int_{z_1}^1 K_{4,1,2}(z_1,z_2)\prod_{i=1}^{2}\left[x(z_1) - x_{i-1}(z_1)\right]\prod_{i=3}^{4}\left[x(z_2) - x_{i-1}(z_2)\right]\ dz_2dz_1 +$$

$$+\int_0^1\int_{z_1}^1 K_{4,1,3}(z_1,z_2)\prod_{i=1}^{3}\left[x(z_1) - x_{i-1}(z_1)\right]\left[x(z_2) - x_3(z_2)\right]\ dz_2dz_1 +$$

$$+\int_0^1 K_{4,1,4}(z_1)\prod_{i=1}^{4}\left[x(z_1) - x_{i-1}(z_1)\right]\ dz_1 +$$

$$+\int_0^1\int_{z_1}^1 K_{4,1,5}(z_1,z_2)\prod_{i=1}^{2}\left[x(z_1) - x_{i-1}(z_1)\right]\prod_{i=3}^{4}\left[x(z_2) - x_{i-1}(z_2)\right]\ dz_2dz_1 +$$

$$+\int_0^1 K_{4,1,6}(z_1)\prod_{i=1}^{4}[x(z_1)-x_{i-1}(z_1)]\ dz_1+$$

$$+\int_0^1\int_{z_1}^1\int_{z_2}^1 K_{4,1,7}\left(\overrightarrow{z}^3\right)[x(z_1)-x_0(z_1)]\times$$

$$\times\prod_{i=2}^{3}[x(z_2)-x_{i-1}(z_2)]\ [x(z_3)-x_3(z_3)]\,dz_3dz_2dz_1+$$

$$+\int_0^1\int_{z_1}^1 K_{4,1,8}(z_1,z_2)[x(z_1)-x_0(z_1)]\prod_{i=2}^{4}[x(z_2)-x_{i-1}(z_2)]\ dz_2dz_1+$$

$$+\int_0^1\int_{z_1}^1 K_{4,1,9}(z_1,z_2)[x(z_1)-x_0(z_1)]\prod_{i=2}^{3}[x(z_1)-x_{i-1}(z_1)][x(z_2)-x_3(z_2)]\ dz_2dz_1+$$

$$+\int_0^1 K_{4,1,10}(z_1)\prod_{i=1}^{4}[x(z_1)-x_{i-1}(z_1)]\ dz_1.$$

The following statement holds true

Theorem 2.1 *For polynomial* (2.1) *to be interpolation for functional* $F(x(\cdot))$ *on continual knot* $x^4\left(\cdot,\overrightarrow{\xi}^4\right)\ \forall\ \overrightarrow{\xi}^4\in\overline{\Omega}_4$ *under corresponding smothness conditions on functional* $F(x(\cdot))$ *it is necessary and sufficient that it's kernels be defined by the following formulas*

$$K_{4,0,0}\left(\vec{z}^4\right)=\prod_{i=1}^{4}[x_i(z_i)-x_{i-1}(z_i)]^{-1}\frac{\partial^4}{\partial z_1\dots\partial z_4}F\left(x^4\left(\cdot,\vec{z}^4\right)\right)\ , \tag{2.2}$$

$$K_{4,0,1}\left(\overrightarrow{z}^3\right)=\prod_{i=1}^{3}[x_i(z_i)-x_{i-1}(z_i)]^{-1}\cdot[x_4(z_3)-x_3(z_3)]^{-1}\times$$

$$\times\left[\left.\frac{\partial^3F(x^4(\cdot,\overrightarrow{z}^4))}{\partial z_3\partial z_2\partial z_1}\right|_{z_4=z_3}-\frac{x_3(z_3)-x_2(z_3)}{x_4(z_3)-x_2(z_3)}\cdot\frac{\partial^3F(x^4(\cdot,(z_1,z_2,z_3,z_3)^T))}{\partial z_3\partial z_2\partial z_1}\right],$$

$$K_{4,0,2}\left(\overrightarrow{z}^2\right)=\prod_{i=1}^{2}[x_i(z_i)-x_{i-1}(z_i)]^{-1}\cdot\prod_{i=3}^{4}[x_4(z_2)-x_{i-1}(z_2)]^{-1}\times$$

$$\times\left[\frac{x_2(z_2)-x_1(z_2)}{x_4(z_2)-x_1(z_2)}\cdot\frac{\partial^2F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_2\partial z_1}-\left.\frac{\partial^2F(x^4(\cdot,(z_1,z_2,z_3,z_3)^T))}{\partial z_2\partial z_1}\right|_{z_3=z_2}\right],$$

$$K_{4,0,3}(z_1)=[x_1(z_1)-x_0(z_1)]^{-1}\prod_{i=1}^{3}[x_4(z_1)-x_i(z_1)]^{-1}\times$$

$$\times\left[\left.\frac{\partial F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1}\right|_{z_2=z_1}-\frac{x_1(z_1)-x_0(z_1)}{x_4(z_1)-x_0(z_1)}\frac{\partial F(x^4(\cdot,(z_1,z_1,z_1,z_1)^T))}{\partial z_1}\right],$$

$$K_{4,1,1}\left(\overrightarrow{z}^3\right) = \prod_{i=1}^{2}[x_i(z_1)-x_{i-1}(z_1)]^{-1}\cdot\prod_{i=3}^{4}[x_i(z_{i-1})-x_{i-1}(z_{i-1})]^{-1}\times$$

$$\times\left[\left.\frac{\partial^3 F(x^4(\cdot,(z_1,t,z_2,z_3)^T))}{\partial z_3\partial z_2\partial z_1}\right|_{t=z_1} - \frac{x_1(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\frac{\partial^3 F(x^4(\cdot,(z_1,z_1,z_2,z_3)^T))}{\partial z_3\partial z_2\partial z_1}\right],$$

$$K_{4,1,2}\left(\overrightarrow{z}^2\right) = \prod_{i=1}^{2}[x_i(z_1)-x_{i-1}(z_1)]^{-1}\cdot\prod_{i=3}^{4}[x_i(z_2)-x_{i-1}(z_2)]^{-1}\times$$

$$\times\left[\left.\frac{\partial^2 F(x^4(\cdot,(z_1,t,z_2,z_3)^T))}{\partial z_2\partial z_1}\right|_{\substack{t=z_1\\ z_3=z_2}} - \frac{x_1(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_1,z_2,z_3)^T))}{\partial z_2\partial z_1}\right|_{z_3=z_2}\right],$$

$$K_{4,1,3}\left(\overrightarrow{z}^2\right) = \prod_{i=1}^{3}[x_i(z_1)-x_{i-1}(z_1)]^{-1}[x_4(z_2)-x_3(z_2)]^{-1}\times$$

$$\times\left[\left.\frac{\partial^2 F(x^4(\cdot,(z_1,t,t,z_2)^T))}{\partial z_2\partial z_1}\right|_{t=z_1} - \frac{x_1(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_1,t,z_2)^T))}{\partial z_2\partial z_1}\right|_{t=z_1}\right],$$

$$K_{4,1,4}(z_1) = \prod_{i=1}^{4}[x_i(z_1)-x_{i-1}(z_1)]^{-1}\times$$

$$\times\left[\left.\frac{\partial F(x^4(\cdot,(z_1,t,t,t)^T))}{\partial z_1}\right|_{t=z_1} - \frac{x_1(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_1,t,t)^T))}{\partial z_1}\right|_{t=z_1}\right],$$

$$K_{4,1,5}(z_1,z_2) = \prod_{i=1}^{2}[x_i(z_1)-x_{i-1}(z_1)]^{-1}\cdot\prod_{i=3}^{4}[x_4(z_2)-x_{i-1}(z_2)]^{-1}\times$$

$$\times\left[\frac{x_1(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\frac{\partial^2 F(x^4(\cdot,(z_1,z_1,z_2,z_2)^T))}{\partial z_2\partial z_1} - \left.\frac{\partial^2 F(x^4(\cdot,(z_1,t,z_2,z_2)^T))}{\partial z_2\partial z_1}\right|_{t=z_1}\right],$$

$$K_{4,1,6}(z_1) = \prod_{i=1}^{4}[x_i(z_1)-x_{i-1}(z_1)]^{-1}\times$$

$$\times\left[\frac{x_1(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\left.\frac{\partial F(x^4(\cdot,(z_1,z_1,t,t)^T))}{\partial z_1}\right|_{t=z_1} - \left.\frac{\partial F(x^4(\cdot,(z_1,t,t,t)^T))}{\partial z_1}\right|_{t=z_1}\right],$$

$$K_{4,1,7}\left(\overrightarrow{z}^3\right) = \prod_{i=1}^{2}[x_i(z_1)-x_{i-1}(z_1)]^{-1}\cdot\prod_{i=3}^{4}[x_i(z_{i-1})-x_{i-1}(z_{i-1})]^{-1}\times$$

$$\times\left[\left.\frac{\partial^3 F(x^4(\cdot,(z_1,z_2,t,z_3)^T))}{\partial z_3\partial z_2\partial z_1}\right|_{t=z_2} - \frac{x_2(z_2)-x_1(z_2)}{x_3(z_2)-x_1(z_2)}\frac{\partial^3 F(x^4(\cdot,(z_1,z_2,z_2,z_3)^T))}{\partial z_3\partial z_2\partial z_1}\right],$$

$$K_{4,1,8}\left(\overrightarrow{z}^2\right) = \prod_{i=1}^{2}[x_i(z_i)-x_{i-1}(z_i)]^{-1}\cdot\prod_{i=3}^{4}[x_i(z_2)-x_{i-1}(z_2)]^{-1}\times$$

$$\times\left[\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,t,t)^T))}{\partial z_2\partial z_1}\right|_{t=z_2}-\frac{x_2(z_2)-x_1(z_2)}{x_3(z_2)-x_1(z_2)}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,z_3)^T))}{\partial z_2\partial z_1}\right|_{z_3=z_2}\right],$$

$$K_{4,1,9}\left(\overrightarrow{z}^2\right)=[x_1(z_1)-x_0(z_1)]^{-1}\prod_{i=1}^{2}[x_3(z_1)-x_i(z_1)]^{-1}[x_4(z_2)-x_3(z_2)]^{-1}\times$$

$$\times\left[\frac{x_1(z_1)-x_0(z_1)}{x_3(z_1)-x_0(z_1)}\frac{\partial^2 F(x^4(\cdot,(z_1,z_1,z_1,z_2)^T))}{\partial z_2\partial z_1}-\left.\frac{\partial^2 F(x^4(\cdot,(z_1,t,t,z_2)^T))}{\partial z_2\partial z_1}\right|_{t=z_1}\right],$$

$$K_{4,1,10}\left(z_1\right)=[x_1(z_1)-x_0(z_1)]^{-1}\prod_{i=1}^{2}[x_3(z_1)-x_i(z_1)]^{-1}[x_4(z_1)-x_3(z_1)]^{-1}\times$$

$$\times\left[\frac{x_1(z_1)-x_0(z_1)}{x_3(z_1)-x_0(z_1)}\left.\frac{\partial F(x^4(\cdot,(z_1,z_1,z_1,t)^T))}{\partial z_1}\right|_{t=z_1}-\left.\frac{\partial F(x^4(\cdot,(z_1,t,t,t)^T))}{\partial z_1}\right|_{t=z_1}\right].$$

Proof. First let us prove necessary condition. We substitute continual knot $x^4(z,\overrightarrow{\xi}^4)$ into formula (2.1) and prove that interpolationness condition $P_4^I(x^4(\cdot,\vec{\xi}^4))=F(x^4(\cdot,\vec{\xi}^4))$ fulfills. We have

$$p^I_{4,0,0}(x^4(\cdot,\overrightarrow{\xi}^4))=\left(\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{\xi_4}\int_{\xi_4}^{1}+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_4}^{1}\int_{z_3}^{1}+\int_{\xi_2}^{\xi_3}\int_{z_1}^{\xi_3}\int_{\xi_3}^{\xi_4}\int_{\xi_4}^{1}+\int_{\xi_1}^{\xi_2}\int_{\xi_3}^{\xi_4}\int_{z_2}^{\xi_4}\int_{\xi_4}^{1}+\int_{\xi_1}^{\xi_2}\int_{\xi_3}^{\xi_4}\int_{\xi_4}^{1}\int_{z_3}^{1}+\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{\xi_4}\int_{z_2}^{\xi_4}\int_{\xi_4}^{1}+\right.$$

$$\left.+\int_{\xi_1}^{\xi_2}\int_{\xi_4}^{1}\int_{z_2}^{1}\int_{z_3}^{1}+\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{\xi_4}\int_{\xi_4}^{1}\int_{z_3}^{1}+\int_{\xi_2}^{\xi_3}\int_{z_1}^{\xi_3}\int_{\xi_4}^{1}\int_{z_3}^{1}+\int_{\xi_3}^{\xi_4}\int_{z_1}^{\xi_4}\int_{z_2}^{\xi_4}\int_{\xi_4}^{1}+\int_{\xi_2}^{\xi_3}\int_{\xi_4}^{1}\int_{z_2}^{1}\int_{z_3}^{1}+\int_{\xi_3}^{\xi_4}\int_{z_1}^{\xi_4}\int_{\xi_4}^{1}\int_{z_3}^{1}+\int_{\xi_3}^{\xi_4}\int_{\xi_4}^{1}\int_{z_2}^{1}\int_{z_3}^{1}+\int_{\xi_4}^{1}\int_{z_1}^{1}\int_{z_2}^{1}\int_{z_3}^{1}\right)$$

$$\prod_{i=1}^{4}\frac{\left[x^4\left(\cdot,\overrightarrow{z}^4\right)-x_{i-1}\left(z_i\right)\right]}{\left[x_i\left(z_i\right)-x_{i-1}\left(z_i\right)\right]}\frac{\partial^4}{\partial z_1\ldots\partial z_4}F\left(x^4\left(\cdot,\vec{z}^4\right)\right)\,dz_i=-p^I_{3,0,0}(x^4(\cdot,\overrightarrow{\xi}^4))+$$

$$+\left(-\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{\xi_4}\frac{\partial^3 F(x^4(\cdot,(z_1,z_2,z_3,\xi_4)^T))}{\partial z_1\partial z_2\partial z_3}-\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_4}^{1}\left.\frac{\partial^3 F(x^4(\cdot,\overrightarrow{z}^4))}{\partial z_1\partial z_2\partial z_3}\right|_{z_4=z_3}-\right. \qquad (2.3)$$

$$-\int_{\xi_2}^{\xi_3}\int_{z_1}^{\xi_3}\int_{\xi_3}^{\xi_4}\frac{\partial^3 F(x^4(\cdot,(z_1,z_2,z_3,\xi_4)^T))}{\partial z_1\partial z_2\partial z_3}-\int_{\xi_1}^{\xi_2}\int_{\xi_3}^{\xi_4}\int_{z_2}^{\xi_4}\frac{\partial^3 F(x^4(\cdot,(z_1,z_2,z_3,\xi_4)^T))}{\partial z_1\partial z_2\partial z_3}-$$

$$-\int_{\xi_1}^{\xi_2}\int_{\xi_3}^{\xi_4}\int_{\xi_4}^{1}\left.\frac{\partial^3 F(x^4(\cdot,\overrightarrow{z}^4))}{\partial z_1\partial z_2\partial z_3}\right|_{z_4=z_3}-\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{\xi_4}\int_{z_2}^{\xi_4}\frac{\partial^3 F(x^4(\cdot,(z_1,z_2,z_3,\xi_4)^T))}{\partial z_1\partial z_2\partial z_3}-$$

$$-\int_{\xi_1}^{\xi_2}\int_{\xi_4}^{1}\int_{z_2}^{1}\left.\frac{\partial^3 F(x^4(\cdot,\overrightarrow{z}^4))}{\partial z_1\partial z_2\partial z_3}\right|_{z_4=z_3}-\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{\xi_4}\int_{\xi_4}^{1}\left.\frac{\partial^3 F(x^4(\cdot,\overrightarrow{z}^4))}{\partial z_1\partial z_2\partial z_3}\right|_{z_4=z_3}-$$

$$-\int_{\xi_2}^{\xi_3}\int_{z_1}^{\xi_3}\int_{\xi_4}^{1}\left.\frac{\partial^3 F(x^4(\cdot,\overrightarrow{z}^4))}{\partial z_1\partial z_2\partial z_3}\right|_{z_4=z_3}-\int_{\xi_3}^{\xi_4}\int_{z_1}^{\xi_4}\int_{z_2}^{\xi_4}\frac{\partial^3 F(x^4(\cdot,(z_1,z_2,z_3,\xi_4)^T))}{\partial z_1\partial z_2\partial z_3}-$$

$$-\int_{\xi_2}^{\xi_3}\int_{\xi_4}^{1}\int_{z_2}^{1}\left.\frac{\partial^3 F(x^4(\cdot,\overrightarrow{z}^4))}{\partial z_1\partial z_2\partial z_3}\right|_{z_4=z_3}-\int_{\xi_3}^{\xi_4}\int_{z_1}^{\xi_4}\int_{\xi_4}^{1}\left.\frac{\partial^3 F(x^4(\cdot,\overrightarrow{z}^4))}{\partial z_1\partial z_2\partial z_3}\right|_{z_4=z_3}-$$

$$-\int_{\xi_3}^{\xi_4}\int_{\xi_4}^{1}\int_{z_2}^{1}\left.\frac{\partial^3 F(x^4(\cdot,\overrightarrow{z}^4))}{\partial z_1\partial z_2\partial z_3}\right|_{z_4=z_3}-\int_{\xi_4}^{1}\int_{z_1}^{1}\int_{z_2}^{1}\left.\frac{\partial^3 F(x^4(\cdot,\overrightarrow{z}^4))}{\partial z_1\partial z_2\partial z_3}\right|_{z_4=z_3}\Bigg)\times$$

$$\times\prod_{i=1}^{3}\frac{\left[x^4\left(\cdot,\overrightarrow{z}^4\right)-x_{i-1}\left(z_i\right)\right]}{\left[x_i\left(z_i\right)-x_{i-1}\left(z_i\right)\right]}\,dz_i.$$

Further

$$p^I_{4,0,1}(x^4(\cdot,\overrightarrow{\xi}^4))=\Bigg(\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_4}^{1}+\int_{\xi_1}^{\xi_2}\int_{\xi_3}^{\xi_4}\int_{\xi_4}^{1}+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_4}^{1}\int_{z_2}^{1}+\int_{\xi_2}^{\xi_3}\int_{z_1}^{\xi_3}\int_{\xi_4}^{1}+\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{\xi_4}\int_{\xi_4}^{1}+\int_{\xi_2}^{\xi_3}\int_{\xi_4}^{1}\int_{z_2}^{1}+\int_{\xi_3}^{\xi_4}\int_{z_1}^{\xi_4}\int_{\xi_4}^{1}+\int_{\xi_3}^{\xi_4}\int_{\xi_4}^{1}\int_{z_2}^{1}+\tag{2.4}$$

$$+\int_{\xi_4}^{1}\int_{z_1}^{1}\int_{z_2}^{1}\Bigg)\left[K_{4,1,1}\left(\overrightarrow{z}^3\right)\right]\times\prod_{i=1}^{3}\frac{\left[x^4\left(\cdot,\overrightarrow{z}^4\right)-x_{i-1}\left(z_i\right)\right]}{\left[x_i\left(z_i\right)-x_{i-1}\left(z_i\right)\right]}\,dz_i.$$

Let us find sum of (2.3) and (2.4)

$$p^I_{4,0,0}(x^4(\cdot,\overrightarrow{\xi}^4))+p^I_{4,0,1}(x^4(\cdot,\overrightarrow{\xi}^4))=-p^I_{3,0}\left(x^4\left(\cdot,\overrightarrow{\xi}^4\right)\right)-p^I_{2,0}\left(x^4\left(\cdot,\overrightarrow{\xi}^4\right)\right)+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,\xi_3,\xi_4)^T))}{\partial z_1\partial z_2}dz_2dz_1+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_3}^{\xi_4}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_3,\xi_4)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_3\left(z_2\right)-x_1\left(z_2\right)}{x_2\left(z_2\right)-x_1\left(z_2\right)}dz_2dz_1+$$

$$+\int_{\xi_2}^{\xi_3}\int_{z_1}^{\xi_3}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,\xi_3,\xi_4)^T))}{\partial z_1\partial z_2}\frac{x_2\left(z_1\right)-x_0\left(z_1\right)}{x_1\left(z_1\right)-x_0\left(z_1\right)}dz_2dz_1-$$

$$+\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{\xi_4}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_3,\xi_4)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_2\left(z_1\right)-x_0\left(z_1\right)}{x_1\left(z_1\right)-x_0\left(z_1\right)}\frac{x_3\left(z_2\right)-x_1\left(z_2\right)}{x_2\left(z_2\right)-x_1\left(z_2\right)}dz_2dz_1-$$

$$+\int_{\xi_3}^{\xi_4}\int_{z_1}^{\xi_4}\left.\frac{\partial^3 F(x^4(\cdot,(z_1,z_2,z_3,\xi_4)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_3\left(z_1\right)-x_0\left(z_1\right)}{x_1\left(z_1\right)-x_0\left(z_1\right)}\frac{x_3\left(z_2\right)-x_1\left(z_2\right)}{x_2\left(z_2\right)-x_1\left(z_2\right)}dz_2dz_1+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_4}^{1}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_3,z_3)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_4\left(z_1\right)-x_1\left(z_1\right)}{x_2\left(z_1\right)-x_1\left(z_1\right)}dz_2dz_1+$$

$$+\int_{\xi_2}^{\xi_3}\int_{\xi_4}^{1}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_3,z_3)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_2\left(z_1\right)-x_0\left(z_1\right)}{x_1\left(z_1\right)-x_0\left(z_1\right)}\frac{x_4\left(z_2\right)-x_1\left(z_2\right)}{x_2\left(z_2\right)-x_1\left(z_2\right)}dz_2dz_1+$$

$$+\int\limits_{\xi_3}^{\xi_4}\int\limits_{\xi_4}^{1}\left.\frac{\partial^3 F(x^4(\cdot,(z_1,z_2,z_3,z_3)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_4(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}dz_2dz_1+$$

$$+\int\limits_{\xi_4}^{1}\int\limits_{z_1}^{1}\left.\frac{\partial^3 F(x^4(\cdot,(z_1,z_2,z_3,z_3)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_4(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_4(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}dz_2dz_1.$$

Write down two more items $p^I_{4,0,2}(x^4(\cdot,\overrightarrow{\xi}^4))$ and $p^I_{4,0,3}(x^4(\cdot,\overrightarrow{\xi}^4))$.

$$p^I_{4,0,2}(x^4(\cdot,\overrightarrow{\xi}^4))=\int\limits_{\xi_1}^{\xi_2}\int\limits_{\xi_4}^{1}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1\partial z_2}dz_2dz_1-$$

$$-\int\limits_{\xi_1}^{\xi_2}\int\limits_{\xi_4}^{1}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_3,z_3)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_4(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}dz_2dz_1+$$

$$+\int\limits_{\xi_2}^{\xi_3}\int\limits_{\xi_4}^{1}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1\partial z_2}dz_2dz_1-$$

$$-\int\limits_{\xi_2}^{\xi_3}\int\limits_{\xi_4}^{1}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_4(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_3,z_3)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}dz_2dz_1+$$

$$+\int\limits_{\xi_3}^{\xi_4}\int\limits_{\xi_4}^{1}\frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1\partial z_2}dz_2dz_1-$$

$$-\int\limits_{\xi_3}^{\xi_4}\int\limits_{\xi_4}^{1}\frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_4(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_3,z_3)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}dz_2dz_1+$$

$$+\int\limits_{\xi_4}^{1}\int\limits_{z_1}^{1}\frac{x_4(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1\partial z_2}dz_2dz_1-$$

$$-\int\limits_{\xi_4}^{1}\int\limits_{z_1}^{1}\frac{x_4(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_4(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_3,z_3)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}dz_2dz_1.$$

$$p^I_{4,0,3}(x^4(\cdot,\overrightarrow{\xi}^4))=\int\limits_{\xi_4}^{1}\frac{x_4(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\left.\frac{\partial F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1}\right|_{z_2=z_1}dz_1-$$

$$-\int\limits_{\xi_4}^{1}\frac{\partial F(x^4(\cdot,(z_1,z_1,z_1,z_1)^T))}{\partial z_1}dz_1.$$

Now we can find

$$\sum_{k=0}^{3}p^I_{4,0,k}(x(\cdot))=-p^I_{3,0}\left(x^4\left(\cdot,\overrightarrow{\xi}^4\right)\right)-p^I_{2,0}\left(x^4\left(\cdot,\overrightarrow{\xi}^4\right)\right)+\tag{2.5}$$

$$+\int\limits_{\xi_1}^{\xi_2}\int\limits_{\xi_2}^{\xi_3}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,\xi_3,\xi_4)^T))}{\partial z_1\partial z_2}dz_2dz_1+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_3}^{\xi_4}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_3,\xi_4)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}dz_2dz_1+$$

$$+\int_{\xi_2}^{\xi_3}\int_{z_1}^{\xi_3}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,\xi_3,\xi_4)^T))}{\partial z_1\partial z_2}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}dz_2dz_1+$$

$$+\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{\xi_4}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_3,\xi_4)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}dz_2dz_1+$$

$$+\int_{\xi_3}^{\xi_4}\int_{z_1}^{\xi_4}\left.\frac{\partial^3 F(x^4(\cdot,(z_1,z_2,z_3,\xi_4)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}dz_2dz_1+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_4}^{1}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1\partial z_2}dz_2dz_1+$$

$$+\int_{\xi_2}^{\xi_3}\int_{\xi_4}^{1}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1\partial z_2}dz_2dz_1+$$

$$+\int_{\xi_3}^{\xi_4}\int_{\xi_4}^{1}\frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1\partial z_2}dz_2dz_1+$$

$$+\int_{\xi_4}^{1}\int_{z_1}^{1}\frac{x_4(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1\partial z_2}dz_2dz_1+$$

$$+\int_{\xi_4}^{1}\frac{x_4(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\left.\frac{\partial F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1}\right|_{z_2=z_1}dz_1-$$

$$-\int_{\xi_4}^{1}\frac{\partial F(x^4(\cdot,(z_1,z_1,z_1,z_1)^T))}{\partial z_1}dz_1.$$

Further we proceed analogously and find the following

1. $p^I_{4,1,1}(x^4(\cdot,\overrightarrow{\xi}^4))+p^I_{4,1,2}(x^4(\cdot,\overrightarrow{\xi}^4))=-p^I_{3,1}\left(x^4\left(\cdot,\overrightarrow{\xi}^4\right)\right)-$

$$-\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{\xi_4}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,t,z_2,\xi_4)^T))}{\partial z_1\partial z_2}\right|_{t=z_1}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}dz_2dz_1+$$

$$+\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{\xi_4}\frac{\partial^2 F(x^4(\cdot,(z_1,z_1,z_2,\xi_4)^T))}{\partial z_1\partial z_2}dz_2dz_1-$$

$$-\int_{\xi_3}^{\xi_4}\int_{z_1}^{\xi_4}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,t,z_2,\xi_4)^T))}{\partial z_1\partial z_2}\right|_{t=z_1}\frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\cdot\frac{x_3(z_1)-x_1(z_1)}{x_2(z_1)-x_1(z_1)}dz_2dz_1+$$

$$+\int_{\xi_3}^{\xi_4}\int_{z_1}^{\xi_4}\frac{\partial^2 F(x^4(\cdot,(z_1,z_1,z_2,\xi_4)^T))}{\partial z_1\partial z_2}\frac{x_3(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\cdot\frac{x_3(z_1)-x_1(z_1)}{x_2(z_1)-x_1(z_1)}dz_2dz_1.$$

2. $p^I_{4,1,3}(x^4(\cdot,\overrightarrow{\xi}^4)) = -p^I_{3,1,2}(x^4(\cdot,\overrightarrow{\xi}^4)) - p^I_{4,1,4}(x^4(\cdot,\overrightarrow{\xi}^4)) -$

$$-\int_{\xi_3}^{\xi_4} \left.\frac{\partial F(x^4(\cdot,(z_1,t,t,\xi_4)^T))}{\partial z_1}\right|_{t=z_1} \frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} \cdot \frac{x_3(z_1)-x_1(z_1)}{x_2(z_1)-x_1(z_1)} dz_1 +$$

$$+\int_{\xi_3}^{\xi_4} \left.\frac{\partial F(x^4(\cdot,(z_1,z_1,t,\xi_4)^T))}{\partial z_1}\right|_{t=z_1} \frac{x_3(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)} \cdot \frac{x_3(z_1)-x_1(z_1)}{x_2(z_1)-x_1(z_1)} dz_1.$$

3. $p^I_{4,1,5}(x^4(\cdot,\overrightarrow{\xi}^4)) = -p^I_{2,0,1}(x^4(\cdot,\overrightarrow{\xi}^4)) - p^I_{4,1,6}(x^4(\cdot,\overrightarrow{\xi}^4)) -$

$$-\int_{\xi_2}^{\xi_3} \frac{\partial^2 F(x^4(\cdot,(z_1,z_1,\xi_4,\xi_4)^T))}{\partial z_1} dz_1 +$$

$$+\int_{\xi_2}^{\xi_3} \left.\frac{\partial F(x^4(\cdot,(z_1,t,\xi_4,\xi_4)^T))}{\partial z_1}\right|_{t=z_1} \frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} dz_1 -$$

$$-\int_{\xi_3}^{\xi_4} \frac{dF(x^4(\cdot,(z_1,z_1,\xi_4,\xi_4)^T))}{dz_1} \frac{x_3(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)} \cdot \frac{x_3(z_1)-x_1(z_1)}{x_2(z_1)-x_1(z_1)} dz_1 +$$

$$+\int_{\xi_3}^{\xi_4} \left.\frac{\partial F(x^4(\cdot,(z_1,t,\xi_4,\xi_4)^T))}{\partial z_1}\right|_{t=z_1} \frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} \cdot \frac{x_3(z_1)-x_1(z_1)}{x_2(z_1)-x_1(z_1)} dz_1.$$

4. $p^I_{4,1,7}(x^4(\cdot,\overrightarrow{\xi}^4)) + +p^I_{4,1,8}(x^4(\cdot,\overrightarrow{\xi}^4)) =$

$$= p^I_{3,0,1}\left(x^4\left(\cdot,\overrightarrow{\xi}^4\right)\right) + \int_{\xi_1}^{\xi_2}\int_{\xi_3}^{\xi_4} \frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,\xi_4)^T))}{\partial z_1 \partial z_2} dz_2 dz_1 -$$

$$-\int_{\xi_1}^{\xi_2}\int_{\xi_3}^{\xi_4} \left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,t,\xi_4)^T))}{\partial z_1 \partial z_2}\right|_{t=z_2} \frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)} dz_2 dz_1 -$$

$$-\int_{\xi_2}^{\xi_3}\int_{\xi_4}^{1} \left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,t,\xi_4)^T))}{\partial z_1 \partial z_2}\right|_{t=z_2} \frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} \cdot \frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)} dz_2 dz_1 +$$

$$+\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{\xi_4} \frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,\xi_4)^T))}{\partial z_1 \partial z_2} \frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} dz_2 dz_1 -$$

$$-\int_{\xi_3}^{\xi_4}\int_{z_1}^{\xi_4} \left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,t,\xi_4)^T))}{\partial z_1 \partial z_2}\right|_{t=z_2} \frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} \cdot \frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)} dz_2 dz_1 +$$

$$+\int_{\xi_3}^{\xi_4}\int_{z_1}^{\xi_4} \frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,\xi_4)^T))}{\partial z_1 \partial z_2} \frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} \, dz_2 dz_1.$$

5. $p^I_{4,1,9}(x^4(\cdot,\overrightarrow{\xi}^4)) = -\int_{\xi_3}^{\xi_4} \frac{\partial F(x^4(\cdot,(z_1,z_1,z_1,\xi_4)^T))}{\partial z_1} dz_1 +$

$$+\int_{\xi_3}^{\xi_4} \left.\frac{\partial F(x^4(\cdot,(z_1,t,t,\xi_4)^T))}{\partial z_1 \partial z_2}\right|_{t=z_1} \frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} dz_2 dz_1 -$$

$-p^I_{4,1,10}(x^4(\cdot,\overrightarrow{\xi}^4)) - p^I_{3,0,2}(x^4(\cdot,\overrightarrow{\xi}^4))$.

From items 1-5 and from (2.5) it follows that

$p_4^I(x(\cdot)) = -p_{3,0,0}^I\left(x^4\left(\cdot, \overrightarrow{\xi}^4\right)\right) - p_{2,0,0}^I\left(x^4\left(\cdot, \overrightarrow{\xi}^4\right)\right) -$
$-F\left(x^4\left(\cdot, \xi_2, \xi_2, \xi_3, \xi_4\right)\right) + F\left(x^4\left(\cdot, \xi_1, \xi_2, \xi_3, \xi_4\right)\right) + F\left(x^4\left(\xi_4, \xi_4, \xi_4, \xi_4\right)\right) -$
$-p_{3,1,1}^I\left(x^4\left(\cdot, \overrightarrow{\xi}^4\right)\right) + F\left(x^4\left(\cdot, \xi_3, \xi_3, \xi_4, \xi_4\right)\right) - F\left(x^4\left(\cdot, \xi_2, \xi_2, \xi_4, \xi_4\right)\right) -$
$-F\left(x^4\left(\cdot, \xi_3, \xi_3, \xi_3, \xi_4\right)\right) + F\left(x^4\left(\cdot, \xi_2, \xi_2, \xi_3, \xi_4\right)\right) - p_{3,1,2}^I(x^4(\cdot, \overrightarrow{\xi}^4)) -$
$-p_{2,0,1}^I(x^4(\cdot, \overrightarrow{\xi}^4)) - -F\left(x^4\left(\cdot, \xi_3, \xi_3, \xi_4, \xi_4\right)\right) + F\left(x^4\left(\cdot, \xi_2, \xi_2, \xi_4, \xi_4\right)\right) - p_{3,0,1}^I\left(x^4\left(\cdot, \overrightarrow{\xi}^4\right)\right)$
$-F\left(x^4\left(\cdot, \xi_4, \xi_4, \xi_4, \xi_4\right)\right) - p_{1,0,0}^I\left(x^4\left(\cdot, \overrightarrow{\xi}^4\right)\right) - -p_0^I\left(x^4\left(\cdot, \overrightarrow{\xi}^4\right)\right) - p_{3,0,2}^I(x^4(\cdot, \overrightarrow{\xi}^4)) +$
$+F\left(x^4\left(\cdot, \xi_3, \xi_3, \xi_3, \xi_4\right)\right) = -p_0^I\left(x^4\left(\cdot, \overrightarrow{\xi}^4\right)\right) - p_{1,0,0}^I\left(x^4\left(\cdot, \overrightarrow{\xi}^4\right)\right) - p_{2,0,0}^I\left(x^4\left(\cdot, \overrightarrow{\xi}^4\right)\right) -$
$-p_{2,0,1}^I(x^4(\cdot, \overrightarrow{\xi}^4)) - p_{3,0,0}^I\left(x^4\left(\cdot, \overrightarrow{\xi}^4\right)\right) - p_{3,0,1}^I\left(x^4\left(\cdot, \overrightarrow{\xi}^4\right)\right) - p_{3,0,2}^I\left(x^4\left(\cdot, \overrightarrow{\xi}^4\right)\right) -$
$-p_{3,1,1}^I(x^4(\cdot, \overrightarrow{\xi}^4)) - p_{3,1,2}^I(x^4(\cdot, \overrightarrow{\xi}^4)) + F\left(x^4\left(\cdot, \xi_1, \xi_2, \xi_3, \xi_4\right)\right).$

Whence we obtain

$$P_4^I(x^4(\cdot, \vec{\xi}^4)) = F(x^4(\cdot, \vec{\xi}^4)).$$

And this is what we had to prove.

Converse statement. Let us prove necessary conditions. Let the functional polynomial (2.1) be interpolation for the functional $F\left(x\left(\cdot\right)\right)$ on continual knot $x^4\left(z, \overrightarrow{\xi}^4\right)$ $\forall$ $\overrightarrow{\xi}^4 \in \overline{\Omega}_4$. We show that it's kernels are defined by the formulas (2.2).

Let us substitute continual knot $x^4\left(z, (\xi_1, \xi_2, \xi_3, \xi_3)^T\right)$ into both parts of polynomial (2.1) and take into account it's interpolationness. Take only that items which contain ξ_1, ξ_2, ξ_3. If we integrate we discard items which vanish when we differentiate with respect to ξ_3, ξ_2 and ξ_1. We obtain

$$F\left(x^4\left(\cdot, \xi_1, \xi_2, \xi_3, \xi_3\right)\right) = \int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_4}^{1}\int_{z_3}^{1} \frac{\partial^4 F(x^4(\cdot, \overrightarrow{z}^4))}{\partial z_1 \partial z_2 \partial z_3 \partial z_4} \frac{x_4\left(z_3\right) - x_2\left(z_3\right)}{x_3\left(z_3\right) - x_2\left(z_3\right)} \partial z_4 dz_3 dz_2 dz_1 +$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{1} K_{4,0,1}\left(\overrightarrow{z}^3\right) \prod_{i=1}^{3}\left[x\left(z_i\right) - x_{i-1}\left(z_i\right)\right] \cdot \left[x\left(z_3\right) - x_3\left(z_3\right)\right] dz_i = \tag{2.6}$$

$$= -\int_{\xi_3}^{1} \left.\frac{\partial^4 F(x^4(\cdot, (\xi_1, \xi_2, z_3, z_4)))}{\partial z_1 \partial z_2 \partial z_3 \partial z_4}\right|_{z_4 = z_3} \frac{x_4\left(z_3\right) - x_2\left(z_3\right)}{x_3\left(z_3\right) - x_2\left(z_3\right)} dz_1 +$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{1} K_{4,0,1}\left(\overrightarrow{z}^3\right) \prod_{i=1}^{3}\left[x\left(z_i\right) - x_{i-1}\left(z_i\right)\right] \cdot \left[x\left(z_3\right) - x_3\left(z_3\right)\right] dz_i.$$

After we differentiate (2.6) with respect to ξ_3, ξ_2, ξ_1 and substituting ξ_3 for z_3, ξ_2 for z_2 and ξ_1 for z_1 we obtain $K_{4,0,1}\left(\overrightarrow{z}^3\right)$. We proceed analogously to find $K_{4,1,7}\left(\overrightarrow{z}^3\right)$. Substitute continual knot $x^4\left(z, (\xi_1, \xi_2, \xi_2, \xi_3)^T\right)$ into both parts of the polynomial (2.1) and take into account it's interpolationness and $p_{4,0,1}^I(x^4(\cdot, \overrightarrow{\xi}^4))$ which we have already found. We take only those items that contain ξ_1, ξ_2, ξ_3. If we integrate we discard items which vanish when we differentiate with respect to ξ_3, ξ_2 and ξ_1. We obtain

$$F\left(x^4\left(\cdot, \xi_1, \xi_2, \xi_2, \xi_3\right)\right) = p_{3,0,0}^I\left(x^4\left(\cdot, \xi_1, \xi_2, \xi_2, \xi_3\right)\right) - \tag{2.7}$$

$$-\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{z_2}^{\xi_3}\frac{\partial^3 F(x^4(\cdot,(z_1,z_2,z_3,\xi_3)^T))}{\partial z_1\partial z_2\partial z_3}\frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}dz_3dz_2dz_1-$$

$$-\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{1}\left.\frac{\partial^3 F(x^4(\cdot,\overrightarrow{z}^4))}{\partial z_1\partial z_2\partial z_3}\right|_{z_4=z_3}\frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}\frac{x_4(z_3)-x_2(z_3)}{x_3(z_3)-x_2(z_3)}dz_3dz_2dz_1+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{1}K_{4,1,7}\left(\overrightarrow{z}^3\right)\left[x(z_1)-x_0(z_1)\right]\prod_{i=2}^{3}\left[x(z_2)-x_{i-1}(z_2)\right]\ \left[x(z_3)-x_3(z_3)\right]dz_3dz_2dz_1+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{1}\left[K_{4,0,1}\left(\overrightarrow{z}^3\right)\right]\frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}\frac{x_4(z_3)-x_2(z_3)}{x_3(z_3)-x_2(z_3)}dz_3dz_2dz_1=$$

$$=\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_3,\xi_3)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}dz_2dz_1+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{1}K_{4,1,7}\left(\overrightarrow{z}^3\right)\left[x(z_1)-x_0(z_1)\right]\prod_{i=2}^{3}\left[x(z_2)-x_{i-1}(z_2)\right]\ \left[x(z_3)-x_3(z_3)\right]dz_3dz_2dz_1.$$

After one differentiates (2.7) with respect to ξ_3, ξ_2, ξ_1 and changes ξ_3 for z_3, ξ_2 for z_2 and ξ_1 for z_1 one obtains $K_{4,1,7}\left(\overrightarrow{z}^3\right)$. We proceed analogously to find $K_{4,1,1}\left(\overrightarrow{z}^3\right)$. Let us substitute continual knot $x^4\left(z,(\xi_1,\xi_1,\xi_2,\xi_3)^T\right)$ taking into account it's interpolationness into both parts of polynomial (2.1). Besides let us take into account $p^I_{4,0,1}(x^4(\cdot,\overrightarrow{\xi}^4))$ and $p^I_{4,1,7}(x^4(\cdot,\overrightarrow{\xi}^4))$ which we have already found. We take only those items that contain ξ_1,ξ_2,ξ_3. If we integrate we discard items which vanish when we differentiate with respect to ξ_3, ξ_2 and ξ_1.

$$p^I_{4,0,0}\left(x^4\left(\cdot,(\xi_1,\xi_1,\xi_2,\xi_3)^T\right)\right)=$$

$$=-\int_{\xi_1}^{\xi_2}\int_{z_1}^{\xi_2}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,\xi_3,\xi_3)^T))}{\partial z_1\partial z_2}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}dz_2dz_1+$$

$$+\int_{\xi_1}^{\xi_2}\int_{z_1}^{\xi_2}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,\xi_2,\xi_3)^T))}{\partial z_1\partial z_2}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}dz_2dz_1-$$

$$-\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,\xi_3,\xi_3)^T))}{\partial z_1\partial z_2}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}dz_2dz_1+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_3,\xi_3)^T))}{\partial z_1\partial z_2}\right|_{z_3=z_2}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)}dz_2dz_1.$$

$$p_{4,0,1}\left(x^4\left(\cdot,(\xi_1,\xi_1,\xi_2,\xi_3)^T\right)\right)=$$

$$= \int_{\xi_1}^{\xi_2}\int_{z_1}^{\xi_2} \frac{\partial^2 F(x^4(\cdot,(z_1,z_2,\xi_3,\xi_3)^T))}{\partial z_1 \partial z_2} \frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} dz_2 dz_1 +$$

$$+ \int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3} \frac{\partial^2 F(x^4(\cdot,(z_1,z_2,\xi_3,\xi_3)^T))}{\partial z_1 \partial z_2} \frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} \frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)} dz_2 dz_1.$$

$$p^I_{4,1,7}\left(x^4\left(\cdot,(\xi_1,\xi_1,\xi_2,\xi_3)^T\right)\right) =$$

$$= -\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3} \left.\frac{\partial^2 F(x^4(\cdot,(z_1,z_2,t,\xi_3)^T))}{\partial z_1 \partial z_2}\right|_{t=z_2} \frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} \frac{x_3(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)} dz_2 dz_1 +$$

$$+ \int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3} \frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,\xi_3)^T))}{\partial z_1 \partial z_2} \frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} dz_2 dz_1.$$

After summation one obtains

$$F\left(x^4\left(\cdot,\xi_1,\xi_1,\xi_2,\xi_3\right)\right) =$$

$$= -\int_{\xi_1}^{\xi_2} \left.\frac{\partial F(x^4(\cdot,(z_1,z_2,\xi_2,\xi_3)^T))}{\partial z_1}\right|_{z_2=z_1} \frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)} dz_1 + \tag{2.8}$$

$$+ \int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\xi_3}\int_{\xi_3}^{1} K_{4,1,1}\left(\overrightarrow{z}^3\right) \prod_{i=1}^{2}\left[x(z_1)-x_{i-1}(z_1)\right] \prod_{i=3}^{4}\left[x(z_2)-x_{i-1}(z_2)\right] dz_3 dz_2 dz_1.$$

After we diffrentiate (2.8) with respect to ξ_3, ξ_2, ξ_1 and changing ξ_3 for z_3, ξ_2 for z_2 and ξ_1 for z_1 we obtain $K_{4,1,1}\left(\overrightarrow{z}^3\right)$. We proceed analogously. Substitute continual knot $x^4\left(z,(\xi_1,\xi_2,\xi_2,\xi_2)^T\right)$ taking into account it's interpolationness into both parts of polynomial (2.1). Besides we take into account $p^I_{4,0,1}(x^4(\cdot,\overrightarrow{\xi}^4))$, $p^I_{4,1,1}(x^4(\cdot,\overrightarrow{\xi}^4))$ and $p^I_{4,1,7}(x^4(\cdot,\overrightarrow{\xi}^4))$ which we have already found.

We leave only those items which contain ξ_1, ξ_2.

$$0 = \{-F\left(x^4\left(\cdot,\xi_1,\xi_2,\xi_2,\xi_2\right)\right) -$$

$$- \int_{\xi_2}^{1} \left.\frac{\partial F(x^4(\cdot,(\xi_1,z_2,z_3,z_3)^T))}{\partial z_2}\right|_{z_3=z_2} \frac{x_4(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)} dz_2 +$$

$$+ \int_{\xi_1}^{\xi_2}\int_{\xi_2}^{1} K_{4,0,2}(z_1,z_2) \prod_{i=1}^{2}\left[x(z_1)-x_{i-1}(z_1)\right] \prod_{i=3}^{4}\left[x(z_2)-x_{i-1}(z_2)\right]\ dz_2 dz_1\} +$$

$$+\left\{ \int_{\xi_2}^{1} \left.\frac{\partial F(x^4(\cdot,(\xi_1,z_2,t,t)^T))}{\partial z_2}\right|_{t=z_2} \frac{x_4(z_2)-x_1(z_2)}{x_2(z_2)-x_1(z_2)} \frac{x_4(z_2)-x_2(z_2)}{x_3(z_2)-x_2(z_2)} dz_2 - \right. \tag{2.9}$$

$$- \int_{\xi_2}^{1} \left.\frac{\partial F(x^4(\cdot,(z_1,z_2,z_2,z_3)^T))}{\partial z_2}\right|_{z_3=z_2} \frac{x_4(z_2)-x_1(z_2)}{x_3(z_2)-x_1(z_2)} \frac{x_4(z_2)-x_2(z_2)}{x_3(z_2)-x_2(z_2)} dz_2 +$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{1} K_{4,1,8}(z_1,z_2)\left[x(z_1)-x_0(z_1)\right]\prod_{i=2}^{4}\left[x(z_2)-x_{i-1}(z_2)\right]\,dz_2dz_1\Bigg\}.$$

We equal expressions in braces to zero after differentiation of (2.9) with respect to ξ_2 and ξ_1 and changing ξ_2 for z_2 we ξ_1 for z_1 to find $K_{4,0,2}(z_1,z_2)$ and $K_{4,1,8}(z_1,z_2)$.

Further we substitute continual knot $x^4\left(z,(\xi_1,\xi_1,\xi_2,\xi_2)^T\right)$ into both parts of polynomial (2.1) and make allowance to already found kernels and take only those items which contain ξ_1,ξ_2. If we integrate we discard items which vanish when we differentiate with respect to ξ_2 and ξ_1.

$$F\left(x^4(\cdot,\xi_1,\xi_1,\xi_2,\xi_2)\right)=\int_{\xi_1}^{\xi_2}\frac{\partial F(x^4(\cdot,(z_1,\xi_2,\xi_2,\xi_2)^T))}{\partial z_1}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}dz_1-$$

$$-\int_{\xi_1}^{\xi_2}\left.\frac{\partial F(x^4(\cdot,(z_1,z_2,\xi_2,\xi_2)^T))}{\partial z_1}\right|_{z_2=z_1}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}dz_1-$$

$$-\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{1}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,t,z_2,z_3)^T))}{\partial z_1\partial z_2}\right|_{\substack{t=z_1\\ z_3=z_2}}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_4(z_2)-x_1(z_2)}{x_3(z_2)-x_1(z_2)}\,dz_2\,dz_1+$$

$$+\int_{\xi_2}^{1}\left.\frac{\partial F(x^4(\cdot,(\xi_2,\xi_2,z_2,z_3)^T))}{\partial z_2}\right|_{z_3=z_2}\frac{x_4(z_2)-x_2(z_2)}{x_3(z_2)-x_1(z_2)}\,dz_2\,-$$

$$-\int_{\xi_2}^{1}\left.\frac{\partial F(x^4(\cdot,(\xi_1,\xi_1,z_2,z_3)^T))}{\partial z_2}\right|_{z_3=z_2}\frac{x_4(z_2)-x_2(z_2)}{x_3(z_2)-x_1(z_2)}\,dz_2\,-$$

$$-\int_{\xi_1}^{\xi_2}\frac{\partial F(x^4(\cdot,(z_1,\xi_2,\xi_2,\xi_2)^T))}{\partial z_1}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\,dz_1.$$

We leave only that items which contain ξ_1,ξ_2.

$$0=\{-F\left(x^4(\cdot,\xi_1,\xi_1,\xi_2,\xi_2)\right)-$$

$$-\int_{\xi_1}^{\xi_2}\left.\frac{\partial F(x^4(\cdot,(z_1,z_2,\xi_2,\xi_2)^T))}{\partial z_1}\right|_{z_2=z_1}\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}dz_1+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{1}K_{4,1,5}(z_1,z_2)\prod_{i=1}^{2}\left[x(z_1)-x_{i-1}(z_1)\right]\prod_{i=3}^{4}\left[x(z_2)-x_{i-1}(z_2)\right]\,dz_2dz_1\}+ \tag{2.10}$$

$$+\Bigg\{-\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{1}\left.\frac{\partial^2 F(x^4(\cdot,(z_1,t,z_2,z_3)^T))}{\partial z_1\partial z_2}\right|_{\substack{t=z_1\\ z_3=z_2}}\times$$

$$\times\frac{x_2(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_4(z_2)-x_1(z_2)}{x_3(z_2)-x_1(z_2)}dz_2dz_1+$$

$$-\int_{\xi_2}^{1} \left.\frac{\partial F(x^4(\cdot,(\xi_1,\xi_1,z_2,z_3)^T))}{\partial z_2}\right|_{z_3=z_2} \frac{x_4(z_2)-x_2(z_2)}{x_3(z_2)-x_1(z_2)} dz_2+$$

$$\left.+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{1} K_{4,1,2}(z_1,z_2)\prod_{i=1}^{2}[x(z_1)-x_{i-1}(z_1)]\prod_{i=3}^{4}[x(z_2)-x_{i-1}(z_2)]\; dz_2dz_1\right\}.$$

We equal expressions in braces to zero after differentiation of (2.10) with respect to ξ_2 and ξ_1 and change of ξ_2 for z_2, ξ_1 for z_1 to find $K_{4,1,5}(z_1,z_2)$ and $K_{4,1,2}(z_1,z_2)$.

Further we substitute continual knot $x^4\left(z,(\xi_1,\xi_1,\xi_1,\xi_2)^T\right)$ into both parts of polynomial (2.1) and make allowance to already found kernels and take only those items which contain ξ_1,ξ_2. If we integrate we discard items which vanish when we differentiate with respect to ξ_2 and ξ_1.

We obtain

$$0= \{-F\left(x^4\left(\cdot,(\xi_1,\xi_1,\xi_1,\xi_2)^T\right)\right)-\int_{\xi_1}^{\xi_2} \left.\frac{\partial F(x^4(\cdot,(z_1,z_2,z_2,\xi_2)^T))}{\partial z_1}\right|_{z_2=z_1}\frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}dz_1+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{1} K_{4,1,9}(z_1,z_2)[x(z_1)-x_0(z_1)]\prod_{i=2}^{3}[x(z_1)-x_{i-1}(z_1)][x(z_2)-x_3(z_2)]\; dz_2dz_1\}+$$

$$+\{\int_{\xi_1}^{\xi_2} \left.\frac{\partial F(x^4(\cdot,(z_1,t,t,\xi_2)^T))}{\partial z_1}\right|_{t=z_1} \frac{x_3(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_3(z_1)-x_1(z_1)}{x_2(z_1)-x_1(z_1)}dz_1- \tag{2.11}$$

$$-\int_{\xi_1}^{\xi_2} \left.\frac{\partial F(x^4(\cdot,(z_1,z_1,z_2,\xi_2)^T))}{\partial z_1}\right|_{z_2=z_1} \frac{x_3(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\frac{x_3(z_1)-x_1(z_1)}{x_2(z_1)-x_1(z_1)}\,dz_1+$$

$$+\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{1} K_{4,1,3}(z_1,z_2)\prod_{i=1}^{3}[x(z_1)-x_{i-1}(z_1)][x(z_2)-x_3(z_2)]\; dz_2dz_1\}.$$

We equal expressions in braces to zero after differentiation of (2.11) with respect to ξ_2 i ξ_1 and change ξ_2 for z_2, ξ_1 for z_1 to find $K_{4,1,9}(z_1,z_2)$ and $K_{4,1,3}(z_1,z_2)$.

Further we substitute continual knot $x^4\left(z,(\xi_1,\xi_1,\xi_1,\xi_1)^T\right)$ into both parts of polynomial (2.1) and make allowance to kernels that we have already found.

Taking into account interpolationness we obtain

$$0=\left\{-F\left(x^4\left(\cdot,(\xi_1,\xi_1,\xi_1,\xi_1)^T\right)\right)-\right.$$

$$-\int_{\xi_1}^{1} \left.\frac{\partial F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1}\right|_{z_2=z_1} \frac{x_4(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}dz_1+$$

$$\left.-\int_{\xi_1}^{1} K_{4,0,3}(z_1)[x(z_1)-x_0(z_1)]\prod_{i=1}^{3}[x(z_1)-x_i(z_1)]\; dz_1\right\}+ \tag{2.12}$$

$$+\left\{-\int_{\xi_1}^{1}\frac{\partial F(x^4(\cdot,(z_1,z_1,z_2,z_2)^T))}{\partial z_1}\bigg|_{z_2=z_1}\frac{x_4(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\frac{x_4(z_1)-x_1(z_1)}{x_2(z_1)-x_1(z_1)}dz_1+\right.$$

$$+\int_{\xi_1}^{1}\frac{\partial F(x^4(\cdot,(z_1,t,t,t)^T))}{\partial z_1}\bigg|_{t=z_1}\frac{x_4(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_4(z_1)-x_1(z_1)}{x_2(z_1)-x_1(z_1)}dz_1+$$

$$\left.+\int_{\xi_1}^{1}K_{4,1,4}(z_1)\prod_{i=1}^{4}[x(z_1)-x_{i-1}(z_1)]\ dz_1\right\}+$$

$$+\left\{-\int_{\xi_1}^{1}\frac{\partial F(x^4(\cdot,(z_1,t,t,t)^T))}{\partial z_1}\bigg|_{t=z_1}\times\right.$$

$$\times\frac{x_4(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_4(z_1)-x_1(z_1)}{x_2(z_1)-x_1(z_1)}\frac{x_4(z_1)-x_2(z_1)}{x_3(z_1)-x_2(z_1)}dz_1+$$

$$+\int_{\xi_1}^{1}\frac{\partial F(x^4(\cdot,(z_1,z_1,t,t)^T))}{\partial z_1}\bigg|_{t=z_1}\times$$

$$\times\frac{x_4(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\frac{x_4(z_1)-x_1(z_1)}{x_2(z_1)-x_1(z_1)}\frac{x_4(z_1)-x_2(z_1)}{x_3(z_1)-x_2(z_1)}dz_1-$$

$$\left.+\int_{0}^{1}K_{4,1,6}(z_1)\prod_{i=1}^{4}[x(z_1)-x_{i-1}(z_1)]\ dz_1\right\}+$$

$$+\left\{-\int_{\xi_1}^{1}\frac{\partial F(x^4(\cdot,(z_1,z_1,z_1,z_2)^T))}{\partial z_1}\bigg|_{z_2=z_1}\times\right.$$

$$\times\frac{x_4(z_1)-x_0(z_1)}{x_3(z_1)-x_0(z_1)}\frac{x_4(z_1)-x_1(z_1)}{x_3(z_1)-x_1(z_1)}\frac{x_4(z_1)-x_2(z_1)}{x_3(z_1)-x_2(z_1)}dz_1+$$

$$+\int_{\xi_1}^{1}\frac{\partial F(x^4(\cdot,(z_1,t,t,t)^T))}{\partial z_1}\bigg|_{t=z_1}\times$$

$$\times\frac{x_4(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}\frac{x_4(z_1)-x_1(z_1)}{x_3(z_1)-x_1(z_1)}\frac{x_4(z_1)-x_2(z_1)}{x_3(z_1)-x_2(z_1)}dz_1+$$

$$\left.-\int_{0}^{1}K_{4,1,10}(z_1)\prod_{i=1}^{4}[x(z_1)-x_{i-1}(z_1)]\ dz_1\right\}.$$

We equal expressions in braces to zero after differentiation of (2.12) with respect to ξ_1 and change ξ_1 for z_1 to find $K_{4,0,3}(z_1)$, $K_{4,1,4}(z_1)$, $K_{4,1,6}(z_1)$ and $K_{4,1,10}(z_1)$.

Converse statement is proved same as the whole theorem.

Let us find remainder term of interpolation polynomial of the third order not demanding the substitution rule to fulfill.

The following statement is valid.

Theorem 2.2 *The following statement holds true*

$F(x(\cdot))=P_3^I(x(\cdot))+R_3(x(\cdot))$,

where

$R_3(x(\cdot))=p_4^I(x(\cdot))\big|_{x_4(z)=x(z)}$.

Proof. We have

$$p^I_{4,0,0}(x^4(\cdot))\big|_{x_4(z)=x(z)} = \int\limits_0^1\int\limits_{z_1}^1\int\limits_{z_2}^1\int\limits_{z_3}^1 \frac{\partial^4 F(x^4(\cdot,\overrightarrow{z}^4))}{\partial z_1\partial z_2\partial z_3\partial z_4}\prod_{i=1}^{3}\frac{x(z_i)-x_{i-1}(z_i)}{x_i(z_i)-x_{i-1}(z_i)}dz_i =$$
$$= -p^I_{3,0,0}(x^4(\cdot)) - p^I_{4,0,1}(x^4(\cdot))\big|_{x_4(z)=x(z)} -$$
$$-\int\limits_0^1\int\limits_{z_1}^1\int\limits_{z_2}^1 \frac{\partial^3 F(x^4(\cdot,(z_1,z_2,z_3,z_3)^T))}{\partial z_1\partial z_2\partial z_3}\prod_{i=1}^{2}\frac{x(z_i)-x_{i-1}(z_i)}{x_i(z_i)-x_{i-1}(z_i)}dz_i =$$
$$= -p^I_{3,0,0}(x^4(\cdot)) - p^I_{2,0,0}(x^4(\cdot)) - p^I_{4,0,1}(x^4(\cdot))\big|_{x_4(z)=x(z)} - p^I_{4,0,2}(x^4(\cdot))\big|_{x_4(z)=x(z)} +$$
$$+\int\limits_0^1\int\limits_{z_1}^1 \frac{\partial^2 F(x^4(\cdot,(z_1,z_2,z_2,z_2)^T))}{\partial z_1\partial z_2}\frac{x(z_1)-x_0(z_1)}{x_1(z_1)-x_0(z_1)}dz_i =$$
$$= -p^I_{3,0,0}(x^4(\cdot)) - p^I_{1,0,0}(x^4(\cdot)) - p^I_{2,0,0}(x^4(\cdot)) - p^I_{4,0,1}(x^4(\cdot))\big|_{x_4(z)=x(z)} -$$
$$- p^I_{4,0,2}(x^4(\cdot))\big|_{x_4(z)=x(z)} - p^I_{4,0,3}(x^4(\cdot))\big|_{x_4(z)=x(z)} + \int\limits_0^1 \frac{\partial F(x^4(\cdot,(z_1,z_1,z_1,z_1)^T))}{\partial z_1}dz_1 =$$
$$= -p^I_{3,0,0}(x^4(\cdot)) - p^I_{1,0,0}(x^4(\cdot)) - p^I_{2,0,0}(x^4(\cdot)) - p^I_{4,0,1}(x^4(\cdot))\big|_{x_4(z)=x(z)} -$$
$$- p^I_{4,0,2}(x^4(\cdot))\big|_{x_4(z)=x(z)} - p^I_{4,0,3}(x^4(\cdot))\big|_{x_4(z)=x(z)} - F(x_0(\cdot)) + F(x(\cdot)).$$

$$p^I_{4,1,1}(x^4(\cdot))\big|_{x_4(z)=x(z)} = -p^I_{3,1,1}(x^4(\cdot)) - p^I_{4,1,2}(x^4(\cdot))\big|_{x_4(z)=x(z)}.$$
$$p^I_{4,1,7}(x^4(\cdot))\big|_{x_4(z)=x(z)} = -p^I_{3,0,1}(x^4(\cdot)) - p^I_{4,1,8}(x^4(\cdot))\big|_{x_4(z)=x(z)}.$$
$$p^I_{4,1,3}(x^4(\cdot))\big|_{x_4(z)=x(z)} = -p^I_{3,1,2}(x^4(\cdot)) - p^I_{4,1,4}(x^4(\cdot))\big|_{x_4(z)=x(z)}.$$
$$p^I_{4,1,9}(x^4(\cdot))\big|_{x_4(z)=x(z)} = -p^I_{3,0,2}(x^4(\cdot)) - p^I_{4,1,10}(x^4(\cdot))\big|_{x_4(z)=x(z)}.$$
$$p^I_{4,1,5}(x^4(\cdot))\big|_{x_4(z)=x(z)} = -p^I_{2,0,1}(x^4(\cdot)) - p^I_{4,1,6}(x^4(\cdot))\big|_{x_4(z)=x(z)}.$$

We sum all obtained expressions and obtain the following

$$R_3(x(\cdot)) = p^I_4(x(\cdot))\big|_{x_4(z)=x(z)},$$

which we had to prove.

3. Special cases

In the formula (2.1) one can use items which under $x_4(z) = x(z)$ turn into $\{-p^I_{2,0,1}(x(\cdot)) - p^I_{3,0,1}(x(\cdot)) - p^I_{3,0,2}(x(\cdot)) - p^I_{3,1,1}(x(\cdot)) - p^I_{3,1,2}(x(\cdot))\}$ instead of items $\tilde{p}^I_4(x(\cdot)) = \sum_{k=1}^{10} p^I_{4,1,k}(x(\cdot))$. For example

$$\tilde{p}^I_4(x(\cdot)) = -\Bigg\{ \left(-p^I_{4,1,5}(x^4(\cdot)) - p^I_{4,1,6}(x^4(\cdot))\right) + \tag{3.1}$$
$$+\int\limits_0^1\int\limits_{z_1}^1 \left[\frac{x_2(z_2)-x_1(z_2)}{x_3(z_2)-x_1(z_2)}\cdot\frac{\partial^2 F\left(x^3(\cdot,(z_1,z_2,z_2)\,T)\right)}{\partial z_1\partial z_2} - \frac{\partial^2 F(x^3(\cdot,\overrightarrow{z}^3))}{\partial z_1\partial z_2}\bigg|_{z_3=z_2}\right]\times$$
$$\times\prod_{i=1}^{2}\frac{[x(z_i)-x_{i-1}(z_i)]}{[x_i(z_i)-x_{i-1}(z_i)]}\prod_{i=3}^{4}\frac{[x(z_{i-1})-x_{i-1}(z_{i-1})]}{[x_i(z_{i-1})-x_{i-1}(z_{i-1})]}\,dz_2dz_1+$$
$$+\int\limits_0^1\left[\frac{\partial F(x^3(\cdot,(z_1,z_2,z_2)^T))}{\partial z_1}\bigg|_{z_2=z_1} - \frac{x_1(z_1)-x_0(z_1)}{x_3(z_1)-x_0(z_1)}\cdot\frac{\partial F(x^3(\cdot,(z_1,z_1,z_1)^T))}{\partial z_1}\right]\times$$

$$\times \frac{[x(z_1)-x_0(z_1)]}{[x_1(z_1)-x_0(z_1)]}\prod_{i=1}^{2}\frac{[x(z_1)-x_i(z_1)]}{[x_3(z_1)-x_i(z_1)]}\,\frac{[x(z_1)-x_3(z_1)]}{[x_4(z_1)-x_3(z_1)]}dz_1+$$

$$+\int_0^1\int_{z_1}^1\left[\frac{x_1(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\cdot\frac{\partial^2 F\left(x^3\left(\cdot,(z_1,z_1,z_3)^T\right)\right)}{\partial z_1\partial z_3}-\left.\frac{\partial^2 F(x^3(\cdot,\vec{z}^3))}{\partial z_1\partial z_3}\right|_{z_2=z_1}\right]\times$$

$$\times\prod_{i=1}^{2}\frac{[x(z_1)-x_{i-1}(z_1)]}{[x_i(z_1)-x_{i-1}(z_1)]}\prod_{i=3}^{4}\frac{[x(z_3)-x_{i-1}(z_3)]}{[x_i(z_3)-x_{i-1}(z_3)]}\,dz_3dz_1+$$

$$+\int_0^1\left[\frac{x_1(z_1)-x_0(z_1)}{x_2(z_1)-x_0(z_1)}\cdot\left.\frac{\partial F\left(x^3\left(\cdot,(z_1,z_1,z_3)^T\right)\right)}{\partial z_1}\right|_{z_3=z_1}-\left.\frac{\partial F(x^3(\cdot,\vec{z}^3))}{\partial z_1}\right|_{\substack{z_2=z_1\\ z_3=z_1}}\right]\times$$

$$\left.\times\prod_{i=1}^{4}\frac{[x(z_1)-x_{i-1}(z_1)]}{[x_i(z_1)-x_{i-1}(z_1)]}\,dz_1\right\}.$$

Then the following statement holds true

Theorem 3.1 *If $\tilde{p}_4^I(x(\cdot))=\sum_{k=1}^{10}p_{4,1,k}^I(x(\cdot))$ has the form (3.1) in interpolation polynomial (2.1) then it is interpolation for functional $F(x(\cdot))$ on continual knot $x^3\left(\cdot,\vec{\xi}^{\,3}\right)$ $\forall\,\vec{\xi}^{\,3}\in\overline{\Omega}_3$ and on the knot $x_4(z)$ under corresponding smoothness conditions on functional $F(x(\cdot))$.*

Theorem 3.2 *If $\tilde{p}_4^I(x(\cdot))=\sum_{k=1}^{10}p_{4,1,k}^I(x(\cdot))$ has the form (3.1) in interpolation polynomial then the following is valid*

$$F(x(\cdot))=P_3^I(x(\cdot))+R_3(x(\cdot)),$$

where

$$R_3(x(\cdot))=\left.p_4^I(x(\cdot))\right|_{x_4(z)=x(z)}.$$

The proof is done same as in previous theorems.

4. Future plans

Constructed interpolation functional polynomial of the fourth order which does not use substitution rule allows to do generalization on the n-th order. This is the goal of future investigation.

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Bibliography

1. Makarov V. L. Necessary and sufficient conditions of existance of functional inrpolation polynomial on continual set of knots / V. L. Makarov, I. I. Demkiv, B. R. Myhalchuk // RAS Ukraine.– 2003.– № 7.– P. 7-12. (in Russian).
2. Makarov V. L. Interpolation formula of Newton type for non-linear functionals / V. L. Makarov, V. V. Khlobystov // RAS AN USSR.– 1989.– Vol. 307.– № 3.– P. 534-537. (in Russian).
3. Makarov V. L. Interpolation polynomial of Newton type with continual knots / V. L. Makarov, V. V. Khlobystov, J. F. Kashpur, B. R. Myhalchuk // Ukr. mat. zhurnal.– 2003.– Vol. 55.– № 6.– P. 779-790. (in Russian).

4. Sobolevsky P. I. Functional interpolation and some approximation formulas for integrals on Gauss maesure / P. I. Sobolevsky // Izv. AN BSSR.– Ser. Fiz.-Mat. Sci.– 1975.– № 2.– P. 5-12. (in Russian).
5. Ulm S. U. On construction of generalized divided diferences / S. U. Ulm, V. V. Poll // Izv. AN ESSR.– 1969.– Vol. 18.– № 1.– P. 100-102. (in Russian).
6. Janovich L. A. Approximate calculation of continual integrals on Gauss measures / L. A. Janovich.– Minsk: Sci. and tech., 1976.– 384 p. (in Russian).
7. Prenter P. M. Zagrange and Hermite interpolation in Banach spaces / P. M. Prenter // Appr, Theory.– 1971.– Vol. 4.– № 4.– P. 419-432.
8. Ignatenko M. V. Newton type operator interpolation formulas based on interpolation by means of rational function / M. V. Ignatenko, L. A. Yanovich // Computational methods in applied mathematics.– 2002.– Vol. 2.– № 2.– P. 143-152.
9. Porter W. A. Synthesis of polinomic system / W. A. Porter // SIAM J. Math. Anal.– 1980.– Vol. 11.– № 2.– P. 308-315.
10. Filipsson L. Kergin interpolation in Banach spaces / L. Filipsson // J. Appr. Teory.– 2004.– Vol. 127.– P. 108-123.
11. Kergin P. A natural interpolation of C^k functions / P. Kergin // J. Appr. Teory.– 1980.– Vol. 29.– № 4.– P. 278-293.

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