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EVOLUTIONARY SEARCH SYSTEMS IN A STOCHASTIC INTEGRAL FUNCTIONAL MODEL

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Анотація. До серії задач пошуку сигналів у багатоканальних системах зв'язку застосовується еволюційна модель. Будується стохастичний коефіцієнт ефективності та його ергодичне усереднення. Наведені модельні приклади.

Abstract. We use an evolution model to study problems of signal search in multichannel systems. An effectiveness stochastic coefficient and its ergodic averaging is constructed. We also give model examples.

1. Introduction

Search problems arise in different areas of the human activity, which explains that the theory of search, which was originated in the works of B. O. Koopman [14–16], V.I. Arkin [1–3], O. V. Staroverov [8], is developing in many directions, sometimes even independently of each other. To solve optimal search problems, one uses very diverse and, usually, a sufficiently sophisticated mathematical machinery. An attempt to construct a unified theory of search was made by O. Hellman [9]. The theory of optimal search was developed in the works of L. D. Stone [18, 19], K. Iida [13] and others.

2. Evolutionary model of search

We will construct a model for a search of signals in multichannel communication systems. If the search of signals takes a long time, then an optimal search strategy changes, since the stochastic characteristics of the signal transmission change with time. To study such search systems, one needs to use evolution models, which in our case will be the stochastic integral functional model [4, 6].

In [10, 11], the authors study a number of optimal search problems for signals in multichannel communication systems with a stationary effectiveness coefficient. These problems are studied with a unified approach. Each such problem is characterized by given fixed characteristics of the operation of the system, and the characteristics that are to be chosen in an optimal way, that is, to make the corresponding effectiveness coefficient assume an extremal value, fixing other given characteristics. It may happen that the real search systems change their characteristics in the course of operation, being influenced by factors external to the system. These may include the time of the day, the season, the weather, the interest in a particular kind of information, etc.

Consider the case where the characteristics change in a jump-like way. To each possible collection of fixed characteristics, associate a state x of the medium, $x \in X$, and assume that it changes according to a semi-Markov law.

For a number of the problems studied in [10, 11], the process of the search for a fixed x is described by a semi-Markov process denoted by $\eta_x(t)$. The effectiveness of search, $M(t)$, can be represented in time t as

$$M(t) = M_0 + C(x)t,$$

Key words. Evolution model, stochastic integral, functional model.

where $M_0 = M(0)$ is the initial value of the search effectiveness, the velocity $C(x)$, $x \in X$, is a given bounded real-valued function of x, the state of the medium.

Let us now consider the search effectiveness in a semi-Markov medium [4, 6, Sect. 4.1.1]. Assume that the coefficient $C(x)$, which depends on states $x \in X$ that are collections of characteristics of the search system, is switched by a semi-Markov process $\xi(t)$ with the phase space $(X, \mathbf{X})$, where $\mathbf{X}$ is a σ-algebra of measurable subsets of X, and a semi-Markov kernel $Q(t, x, B), x \in X, B \in \mathbf{X}$.

Let the semi-Markov process $\xi(t)$ be defined by a Markov renewal process $(\xi_n, \theta_n; n \geq 0)$, namely, $\xi(t) := \xi_{\nu(t)}$, $\nu(t) = \max\{n : \tau_n \leq t\}$, $\tau_n = \sum\limits_{k=1}^{n} \theta_k$, $n \geq 1$, $\tau_0 = \theta_0 = 0$.

Then the stochastic search effectiveness is given by the integral

$$M_c(t) = M_0 + \int_0^t C(\xi(s))ds,$$

or

$$M_c(t) = M_0 + \sum_{k=1}^{\nu(t)} \theta_k C(\xi_{k-1}) + \gamma(t) C(\xi(t)),$$

where $\gamma(t) = t - \tau_{\nu(t)}$, that is, it is given as a stochastic integral functional.

To simplify the description of the constructed stochastic integral functional, we use the phase lumping algorithm for stochastic evolution systems in a semi-Markov stochastic medium [4, 6, Sect. 4].

We make the following assumptions.

1. The semi-Markov process $\xi(t)$ on the phase space $(X, \mathbf{X})$ is uniformly ergodic [12] with a stationary distribution $\pi(B)$, $B \in \mathbf{X}$, and the stationary distribution $\rho(B)$, $B \in \mathbf{X}$, of the embedded Markov chain $(\xi_n; n \geq 0)$ is related to the distribution $\pi(B)$ by the formula
$$m\pi(dx) = \rho(dx)m(x),$$
where $m := \int\limits_X \rho(dx)m(x)$ and $m(x) = \int\limits_0^\infty (1 - Q(t, x, X))dt$.

2. The evolution of the search effectiveness is considered in a stationary mode, i.e, over significant time intervals so that the ergodicity effect of the semi-Markov process $\xi(t)$ shows in an essential way. In other words, we need to introduce a time scaling coefficient $T > 0$, and to consider the switching semi-Markov process in "fast" changing time, i.e., to introduce a semi-Markov process $\xi_T(t) = \xi(Tt)$, $t \geq 0$, that has a sufficiently large number of switches in a fixed time interval.

Moreover, in order to substantiate the solution algorithm for the search problems under consideration, we assume that the following hold.

3. The time that the semi-Markov process $\xi(t)$ occupies each state exceeds some constant Δ.

4. On each interval θ_n/T, the corresponding search process, which is described by the ergodic semi-Markov process $\eta_x(t)$, is also conducted in the stationary mode, that is, θ_n/T are sufficiently large time intervals, so that the ergodic effects of the search process at a fixed state of the medium shows in an essential way. In order to satisfy this condition, we introduce one more scaling coefficient $S > 0$ and a semi-Markov process $\eta_x(t)$ in the ST-fast time, that is, $\eta_{x,S,T}(t) = \eta_x(STt)$, $t \geq 0$, where the

number of switches of the semi-Markov process $\eta_{x,S,T}(t)$ on the interval θ_n/T for some fixed $T > 0$ is sufficiently large (by Condition 3 for each $n = 1, 2, ..$). We remark that the scaling coefficients do not influence the coefficient $C(x)$.

Then the stochastic search effectiveness with the parameters S, T is given by the integral

$$M_{c,S,T}(t) = M_0 + \int_0^t C(\xi(\tau T))d\tau.$$

The averaged stochastic search effectiveness is given by the relation

$$\widehat{M}(t) = M_0 + \widehat{C}t,$$

where

$$\widehat{C} = \int_X \pi(dx)C(x).$$

It is a uniform motion with constant velocity $\widehat{C}$.

3. Search models for a fixed state of the medium

Let us now pass to the above mentioned series of optimal signal search problems for multichannel connection systems, the problems that can be used to construct the mentioned evolution model.

Let us give a brief setting for one of these problems.

A search with accompanying the sought signals.

Suppose we have N connection channels that transmit signals independently and a single search system. The i-th channel transmits signals e_0 and e_1, and their passage gives rise to a renewal process. The signal e_1 is the sought signal, the signal e_0 is irrelevant. The connection channels are grouped into M classes of one-type signals in the sense that the probability characteristics of the transmitted signals are the same. Namely, the time that the signal e_1 stays in any of the channels of type i has an exponential distribution with parameter $\lambda_i^{(1)}$, and the time for the signal e_0 has, correspondingly, an exponential distribution with parameter $\lambda_i^{(0)}$. The set E_i of the number of channels of type i contains N_i elements.

The search system moves over the channels in a random way with probabilities k_{ij}, $i \neq j$, $k_{ii} = 0$, of switching from channel i to channel j to find the signal e_1 and then follow this type of signals. To distinguish the signal e_0, it is necessary for the search system to stay at channel i and for the signal to be there a deterministic time $t_0^{(i)}$, to recognize the signal e_1, the corresponding deterministic time $t_1^{(i)}$ is needed.

Denote by C the ratio of the expectation of the number of signals e_1 recognized by the search system in all channels in a unit of time in the stationary search mode and the expectation of the number of all signals e_1 that passed in all channels in a unit of time in the stationary search mode. We will call this ratio the search effectiveness coefficient. The search effectiveness at time t is defined by $M(t) = Ct$.

We need to find probabilities k_{ij}, $i \neq j$, of transitions of the search system from channel to channel in such a way that the effectiveness coefficient C would assume a maximal value for the given search parameters $N, t_j^{(i)}, \lambda_j^{(i)}, j = 0, 1,\ i = \overline{1, N}$.

In order to overcome the difficulties that are related to a large number of channels in real communication systems, we use the phase lumping method for large systems [7, 17] when solving the above series of problems.

Finding $C(x)$ for the entire series of problems in the case where the number of channels is large is reduced to solving a mathematical programming problem of the form

$$\begin{cases} \sum_{i=1}^{M} \Phi_i(P_i) \to \max, \\ P_i \geq 0, \quad i = \overline{1, M}, \\ \sum_{i=1}^{M} P_i = 1, \end{cases} \tag{3.1}$$

taking into account some additional conditions in certain cases.

For the entire series of problems, the functions $\Phi_i(P_i)$, $i = \overline{1, M}$, are found analytically and are concave. It is proved that the optimal search strategy is the same for the channels of the same type.

Mathematical programming problems of type 3.1 are solved numerically [11].

4. Model examples

We will give some model examples where some probability characteristics of signal transmission in channels change in time for the search problem considered above. We will consider the stochastic optimal search effectiveness. The time is measured in seconds.

In all examples, we use the following. Let the channels be of three types, that is, $M = 3$, and

$$\lambda_0^{(1)} = \lambda_0^{(2)} = \lambda_0^{(3)} = 1,0 \cdot 10^{-2},$$
$$\lambda_1^{(2)} = 2\lambda_1^{(1)}, \quad \lambda_1^{(3)} = 3\lambda_1^{(1)}.$$

The recognition time for the signals e_o and e_1 is the same for the channels of all types and equals t_0.

Let the semi-Markov kernel for the switching process $\xi(t)$ with a discrete set of states, E, be given as $Q(t, i, j) = p_{ij} G_s(t)$, $i, j \in E$.

Example 4.1 *Let $t_0 = t_0^{(r)}$ depend on the state r of the environment, $r = 1$ for the morning and evening, $r = 2$ for the daytime, $r = 1$ for the evening, $r = 4$ for the night.*

The number of channels of each type is the same and equals $N_1 = N_2 = N_3 = 15$,

$$\lambda_1^{(1)} = 1.0,$$
$$E = \{1, 2, 3, 4\},$$

$G_i(t)$ is a degenerate distribution supported at the point $6.91 \cdot 10^6$ for $i = 2$ (the daytime), at the point $5.18 \cdot 10^6$ for $i = 1, 3$ (the morning or the evening), at the point $3.46 \cdot 10^6$ for $i = 4$ (the night),

$$p_{12} = p_{23} = p_{34} = p_{41} = 1,$$
$$t_0^{(i)} = 5 \cdot 10^{-4} \text{ for } i = 1, 3, \quad t_0^{(4)} = 5 \cdot 10^{-3}, \quad t_0^{(2)} = 5 \cdot 10^{-5}.$$

Then, we have that $C(4) = 0.68$, $C(1) = C(3) = 0.9$, $C(2) = 0.94$, $\widehat{C} = 0.88$.

Example 4.2 *Let the length of the sought signals* $\lambda_1^{(1)} = \lambda_1^{(1)}(r)$ *depend on the state* r *of the environment, i.e., the search goal changes, and*

$$N_1 = N_2 = N_3 = 30, \quad t_0 = 5.0 \cdot 10^{-4},$$
$$E = \{1, 2, 3\},$$
$$\lambda_1^{(1)}(1) = 0.24, \quad \lambda_1^{(1)}(2) = 0.52, \quad \lambda_1^{(1)}(3) = 1.0,$$

$G_i(t)$ *be a uniform distribution on the line segment* $[6.91 \cdot 10^6, \ 8.06 \cdot 10^6]$ *for* $i = 1$, *on the line segment* $[5.18 \cdot 10^6, \ 6.91 \cdot 10^6]$ *for* $i = 2$, *on the line segment* $[3.46 \cdot 10^6, \ 6.91 \cdot 10^6]$ *for* $i = 3$, *correspondingly,*

$$p_{i,1} = 0.5, \quad p_{i,2} = 0.3, \quad p_{i,3} = 0.2, \quad i = 1, 2, 3.$$

Then we have

$$C(1) = 0.64, \quad C(2) = 0.76, \quad C(3) = 0.86, \quad \widehat{C} = 0.71.$$

5. Conclusions

Using the phase lumping algorithm and the averaging algorithm for evolution stochastic systems in a stochastic medium, we proved that the averaged stochastic search effectiveness coefficient for the considered series of problems is a uniform motion with constant velocity $\widehat{C} = \int_X \pi(dx) C(x)$.

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