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APPLICATION OF FINITE ELEMENTS METHOD FOR SOLVING VARIATIONAL PROBLEMS OF CHANNEL FLOWS

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Резюме. Виведено рівняння руху руслового потоку в псевдопризматичному руслі. Побудовано початково-крайову задачу руслового потоку в гідродинамічному наближенні. Сформульовано варіаційну постановку задачі, для якої при дискретизації за просторовою змінною використано метод скінченних елементів з базисними лінійними і квадратичними функціями та при дискретизації за часом – однокрокову рекурентну схему. В умовах рівноваги сил опору і сили земного тяжіння побудовано рівняння кінематичної хвилі, з врахуванням доданку із числом Рейнольдса та другою похідною за просторовою змінною. На тестовому прикладі показано порівняння цих двох підходів з врахуванням зміни градієнтів лінії середнього дна русла.

Abstract. The equation of motion of the channel flow in the pseudo prismatic channel is derived. The initial-boundary value problem of the channel flow in the hydrodynamic approximation is constructed. The variational problem was formulated and solved by method of finite elements with basic linear and quadratic functions for the spatial variable, and at time discretization one step recurrent scheme was constructed. In the conditions of the balance of the forces of resistance and the forces of gravity, the equation of the kinematic wave was derived, taking into account the addition with the number of Reynolds and the second derivative of the spatial variable. The test example shows a comparison of these two approaches, taking into account the change of the gradients of the line of the middle bottom of the channel.

1. Introduction

The transformation of the natural environment and global climate change are causing changes in hydrological systems. The estimation of such changes can be made on the basis of experimental data by comparing the hydrological characteristics before and after anthropogenic impact. However, the possibilities for such estimations are very limited, as the hydro meteorological conditions vary greatly. The main perspectives for the development of research methods and predictions of the behavior of natural hydrological systems are currently solved with the help of their mathematical modeling [1, 13].

In the general study of such an entire system, taking into account all the factors of influence, is a complex and not always appropriate task for study, therefore, only a some part of the region is investigated. The object of research can be the territory of the watershed of the river, which is characterized by

Key words. Variational problem, initial-boundary value problem, Galerkin approximations, channel flow, kinematic and hydrodynamic approximations.

similar climatic conditions and is under the influence of similar factors affecting the movement of fluid. For the description of water streams [2, 5, 10, 11], two approaches are most often used.

One of them is so-called hydrodynamic approach [2, 4, 9], in which the general laws of conservation of momentum, energy, mass are used to describe the processes. In this case, a complicated system of equations is used, usually non-linear, and in many cases, this task is cumbersome to estimate the amount of water.

The second approach is based on the equation of the kinematic wave [3], which are formed in the direction of the flow and occur under conditions of equilibrium of the forces of resistance and forces of gravity. These waves, which mainly affect the formation of the channel flow, which, unlike other types of waves, are formed in different directions and therefore quickly disappear.

In this paper, the flow of water is considered on one of the main elements of the watershed, namely in the inflows and in the main rivers, and these channels will be called pseudoprizmatic. Such channels are formed by moving a curve along a middle bottom line, while it is assumed that the depth of flow is very small compared with the radius of the curvature of the bottom line and the middle line of the free surface is horizontal in any normal section of the flow.

This mathematical model depends on many factors that can change fast enough, so this model must be stable to external and internal influences that significantly modify the solution of the problem. For approximation of the solution linear basic functions were used.

Since the problem is nonlinear, the solution acquires (gets) large positive and negative values, especially in the case of sharp changes of relief of the bottom of the flow. Therefore, the order of approximations of the solution and was shown the feasibility of this approach on different test examples [6].

2. Equation of water flow in pseudoprismatic channel

Choose a coordinate system such that the axis x is directed on the tangent straight to the middle bottom line, and the coordinate lines y and z are straight lines lying in the normal to the bottom of the plane so that y is directed horizontally (Fig. 1).

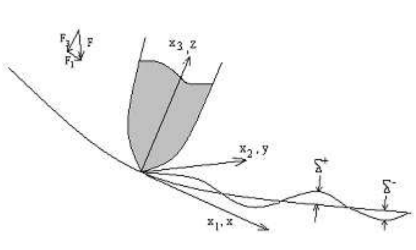

Fig. 1. Form of the channel flow.

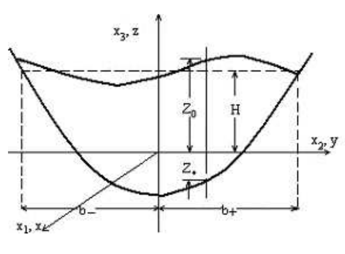

Fig. 2. Cross section of the flow.

The system of equations that characterize the motion of fluid:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0; \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}=X-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}\right); \tag{2}$$

$$\frac{\partial v}{\partial t}+\frac{\partial vu}{\partial x}+\frac{\partial vv}{\partial y}+\frac{\partial vw}{\partial z}=Y-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{1}{\rho}\left(\frac{\partial \tau_{yx}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}\right); \tag{3}$$

$$\frac{\partial w}{\partial t}+\frac{\partial wu}{\partial x}+\frac{\partial wv}{\partial y}+\frac{\partial ww}{\partial z}=Z-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{1}{\rho}\left(\frac{\partial \tau_{zx}}{\partial x}+\frac{\partial \tau_{zy}}{\partial y}+\frac{\partial \tau_{zz}}{\partial z}\right). \tag{4}$$

Equation (1) is the equation of continuity for incompressible fluid, and (2) – (4) the Navier-Stokes equations in which u, v, w and X, Y, Z are projections of the velocity vector v and the vector of acceleration Capacitive forces F on the axis x, y, z.

We integrate the equation (1) with the area of the cross-section of the flow (Fig. 2):

$$\frac{1}{F}\int\limits_{b_-}^{b_+}dy\int\limits_{z_0}^{H}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)dz=0. \tag{5}$$

We use the differentiation formula under the integral sign, and taking into account the symmetry of the channel as to the XOZ plane, when all integrals of F containing $\frac{\partial}{\partial y}$ are equal zero, we obtain

$$\frac{1}{F}\int\limits_{b_-}^{b_+}dy\int\limits_{z_0}^{H}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)dz=0. \tag{6}$$

Since on the surface of the bottom of the flow $z=z_0$ the vector of velocity is zero, then $u_{z=z_0}=0$.

We set the kinematic condition on a free surface:

$$w_{z=H}=\frac{\partial H}{\partial t}+u_{z=H}\frac{\partial H}{\partial x} \tag{7}$$

and the fact that the value $\int\limits_{b_-}^{b_+}\int\limits_{z_0}^{H}udzdy=Q$ is the rate of flow, then equation (6) is written as follows:

$$\frac{\partial Q}{\partial x}+\frac{\partial H}{\partial t}B=0. \tag{8}$$

Let us turn to the equations of motion. It is obvious that for flow in the gravity field $X=g\sin\delta, Y=0, Z=-g\cos\delta=-g_*$, where g-acceleration of gravity, δ – the sharp angle between the horizontal plane and the tangent to the line of the middle bottom. We integrate equation (4) for z and express the value of pressure:

$$\frac{p}{\rho} = \frac{p_H}{\rho} + (H - z_0)g_* - w^2 + \frac{\partial}{\partial t}\int\limits_{z_0}^{H} wdz + \frac{\partial}{\partial x}\int\limits_{z_0}^{H} wudz + \\ + \frac{\partial}{\partial y}\int\limits_{z_0}^{H} wvd\xi + \frac{1}{\rho}\left(\frac{\partial}{\partial x}\int\limits_{z_0}^{H} \tau_{zx}dz + \frac{\partial}{\partial y}\int\limits_{z_0}^{H} \tau_{zy}dz - \tau_{zz}\right), \tag{9}$$

We substitute this value of p into equation (2), and we integrate the result by the area of the cross section F, we obtain:

$$\frac{\partial}{\partial t}\int\limits_{b_-}^{b_+} dy \int\limits_{z_*}^{z_0} udz + \frac{\partial}{\partial x}\int\limits_{b_-}^{b_+} dy \int\limits_{z_*}^{z_0} u^2dz = \\ = g\left(\sin\delta - \frac{\partial z_0}{\partial x}\right)\int\limits_{b_-}^{b_+}\int\limits_{z_*}^{z_0} dzdy - \frac{1}{\rho}\int\limits_{b_-}^{b_+} (\tau_{zx})_{z=z_*}\, dy + \varepsilon. \tag{10}$$

where ε – additions that do not significantly affect the solution of the problem.

We use the expression defined in the hydraulics of turbulent flows

$$\frac{1}{\rho g F}\int\limits_{b_-}^{b_+} (\tau_{zx})_{z=z_*}\, dy = \frac{Q^2}{K^2} = \frac{U^2}{C^2 R}, \tag{11}$$

where $K = CF\sqrt{R}$ – channel capacity; R – hydraulic radius; C – coefficient of Chezy. Then equation (10) will be written as:

$$\frac{1}{g}\left(\frac{\partial U}{\partial t} + U\frac{\partial \alpha U}{\partial x} - \frac{\alpha - 1}{F}U\frac{\partial F}{\partial t}\right) = i - \frac{\partial H}{\partial x} - \frac{U^2}{C^2 R} + \varepsilon. \tag{12}$$

If in (12) we neglect a addition ε, we obtain an hydrodynamic equation of one-dimensional unstable, slowly changing motion.

3. Initial-boundary problem of the channel flow in hydrodynamic approximation

If $Q = UF$, then from (8) follows that equation will be written as:

$$\frac{\partial (UF)}{\partial x} + B\frac{\partial H}{\partial t} = 0$$

From where:

$$\frac{\partial (UF)}{\partial x} + \frac{\partial F}{\partial t} = 0; \tag{13}$$

In equation (12) we neglect by addition ε, then equation will be written:

$$\frac{1}{g}\frac{\partial U}{\partial t} + \frac{\alpha}{g}U\frac{\partial U}{\partial x} - \frac{\alpha - 1}{g}\frac{U}{F}\frac{\partial F}{\partial t} + \frac{1}{B}\frac{\partial F}{\partial x} + \frac{U^2}{C^2 R} = i, \tag{14}$$

where U - flow velocity and F – cross-sectional area; $g = 9{,}8\ m/s^2$ – acceleration of gravity; C=const– coefficient of Chezy; $i = \sin\delta$, where δ – the angle of the midline of the channel bottom to the x-axis; $B = b_+ - b_-$ – width of the channel; R=const – hydraulic radius; α– parameter adjustments of movement.

Complement these equations by initial

$$U\,|_{t=0} = u_0(x), F\,|_{t=0} = f_0(x) \ \ on \ \ [0, L] \tag{15}$$

and boundary conditions

$$U(t, 0) = 0, F(t, 0) = 0. \tag{16}$$

obtain initial-boundary problem of the unknown – the flow velocity U and cross-sectional area F.

So, system of equations (13)–(16) describe initial-boundary problem of fluid flow in open pseudoprizmatic channel.

3.1. **Variational problem.** Choose spaces of allowable functions $H := L^2(\Omega)$, $V := H^1(\Omega)$, where $\Omega = [0,\ \ L]$.

To construct the variational problem multiply equation (13) an arbitrary function $\varphi \in V$, and the (14) – $\psi \in V$ and integrate the results by region Ω.

Input such bilinear form:

$$a(u, f, \varphi) = \int\limits_{\Omega} u\frac{\partial f}{\partial x}\varphi dx;\ \ b(u, \varphi) = \int\limits_{\Omega} u\varphi dx;\ \ c(u, \varphi) = \int\limits_{\Omega} \frac{\partial u}{\partial x}\varphi dx;$$

$$d(u, f, \varphi) = \int\limits_{\Omega} uf\varphi dx;$$

and linear functional

$$l(\varphi) = \int\limits_{\Omega} i\varphi dx.$$

Then variational formulation of initial-boundary problem (13)–(16) can be written as:

$$\begin{cases} Given: \quad u_0, f_0 \in H; \\ Find\ a\ pair: \quad (u, f) \in L^2(0, T; V \times V) \ \ such \ \ that \\ a(u(t), f(t), \varphi) + a(f(t), u(t), \varphi) + b(f'(t), \varphi) = 0; \\ \frac{1}{g}b(u'(t), \psi) + \frac{\alpha}{g}a(u(t), u(t), \psi) + \\ +\frac{1}{B}c(f(t), \psi) + \frac{1}{C^2R}d(u(t), u(t), \psi) - \\ -\frac{\alpha-1}{g}d(w(t), f'(t), \psi) = \langle l, \psi\rangle\,,\ \forall t \in (0,\, T], \\ b(u(0) - u_0, \varphi) = 0, b(f(0) - f_0, \psi) = 0,\ \forall \varphi, \psi \in V. \end{cases} \tag{17}$$

The solution to this problem will be search using the finite elements method.

4. Discretization variation problem in time variable

Divide the length of time [0,T] in $N_T + 1$ equal parts $[t_j, t_{j+1}]$ with length $\Delta t = t_{j+1} - t_j$, $j = 0, ..., N_T$. On each interval $[t_j, t_{j+1}]$ looking solutions of (5). Solutions$u(x,t), f(x,t) \in L^2(0,T;V)$ to this problem approximate by polynomials form

$$\begin{cases} u_{\Delta t}(x,t) = \{1-\omega(t)\}\, u^j(x) + \omega(t) u^{j+1}(x); \\ f_{\Delta t}(x,t) = \{1-\omega(t)\}\, f^j(x) + \omega(t) f^{j+1}(x); \\ t \in [t_{j,} t_{i+1}], j = 0,1,...,N_T - 1, \omega(t_j, t) = \frac{t-t_j}{\Delta t} \end{cases} \quad (18)$$

with unknown functions $u^j(x), f^j(x) \in V_h$.

For functional $l(x,t) \in V_h^1$ in problem (17) will use the following approximation

$$l_{\Delta t}(x,t) = l_{j+1/2} = l(t_{j+1/2}, x). \quad (19)$$

Then recurrent scheme [12, 14] will be written as:

$$\begin{cases} Given : \Delta t, \omega(t) = const > 0,\ u^j, f^j \in V \times V. \\ Find : u^{j+1}, f^{j+1} \in V \times V, such \quad that : \\ b\left(f^{j+1/2}, \varphi\right) + \Delta t \gamma a\left(u^j, f^{j+1/2}, \varphi\right) + \\ +\Delta t \gamma a\left(u^{j+1/2}, f^j, \varphi\right) + \Delta t \gamma a\left(f^{j+1/2}, u^j, \varphi\right) + \\ +\Delta t \gamma a\left(f^j, u^{j+1/2}, \varphi\right) = -a\left(u^j, f^j, \varphi\right) - a\left(f^j, u^j, \varphi\right); \\ \frac{1}{g} b\left(u^{j+1/2}, \psi\right) + \frac{\alpha}{g} \Delta t \beta \left[a\left(u^j, u^{j+1/2}, \psi\right) + a\left(u^{j+1/2}, u^j, \psi\right)\right] + \\ \frac{1}{B} \Delta t \beta c\left(f^{j+1/2}, \psi\right) + \frac{2}{C^2 R} \Delta t \beta d\left(u^j, u^{j+1/2}, \psi\right) - \\ -\frac{\alpha - 1}{g} d(w^j, f^{j+1/2}, \psi) = \\ = \left\langle l_{j+1/2}, \psi\right\rangle - \frac{\alpha}{g} a\left(u^j, u^j, \psi\right) - \frac{1}{B} c\left(f^j, \psi\right) - \frac{1}{C^2 R} d\left(u^j, u^j, \psi\right); \\ u^{j+1} = u^j + \Delta t u^{j+1/2}, f^{j+1} = f^j + \Delta t f^{j+1/2}. \end{cases} \quad (20)$$

The scheme provides that the initial solution (u^0, f^0) defined by initial conditions (16).

5. Discretization of variation problem for spatial variables

Choose a sequence of finite spaces approximations V_h of the space V with properties $\dim V_h \xrightarrow[h \to 0]{} \infty$. Then (u_h, v_h) – semi discrete approximation of solution *(u,f)*.

The interval [0, L] divide using sequence equally spaced units: $x_i = i \cdot h, i = 0, ..., N, h = \frac{L}{N}$ on N finite segments $[x_i, x_{i+1}]\,, i = 0, 1, ..., N-1$.

Choose a base $\{\varphi_j\}_{j=1}^N$, $\{\psi_i\}_{i=1}^M$ in space approximations V_h.

Define functions $u_h^j(x) = \sum_{i=1}^N U_i^j \varphi_i(x)\,, f_h^j(x) = \sum_{i=1}^N F_i^j \varphi_i(x)$ a schedule of for the basis functions $\{\varphi_i\}_{i=1}^N$, $\{\psi_i\}_{i=1}^M$ and unknown coefficients $U = \{U_i\}_{i=1}^M, F = \{F_i\}_{i=1}^N$.

Continuous piecewise defined basis functions $\{\varphi_i(x)\}_{i=1}^N$ of the space V_h chosen as linear polynomials, and $\{\psi_i(x)\}_{i=1}^M$- in the form of quadratic functions.
Functions $\{\varphi_i(x)\}_{i=1}^N$ and $\{\psi_i(x)\}_{i=1}^M$ denote as:

$$\varphi_i(x) = \begin{cases} 0, & 0 \le x \le x_{i-1}, \\ \frac{x-x_{i-1}}{h}, & x_{i-1} \le x \le x_i, \\ \frac{x_{i+1}-x}{h}, & x_i \le x \le x_{i+1}, \\ 0, & x_i \le x \le L. \end{cases}$$

$$\psi_i(x) = \begin{cases} 0, & 0 \le x \le x_{i-2}, \\ \frac{2(x-x_{i-2})(x-x_{i-1})}{h^2}, & x_{i-2} \le x \le x_{i-1}, \\ \frac{4(x-x_{i-1})(x-x_i)}{-h^2}, & x_{i-1} \le x \le x_i, \\ \frac{2(x-x_i)(x-x_{i+1})}{h^2}, & x_i \le x \le x_{i+1}, \\ 0, & x_{i+1} \le x \le L. \end{cases}$$

Overlaid matrices we obtain recurrent scheme as follows[7, 8]:

$$\begin{cases} Given: \ \ \Delta t, \gamma, \beta = const > 0; \ u^j, f^j \in R^n. \\ Find: \ u^{j+1}, f^{j+1} \in R^n, \\ such \ \ that: \\ \left[B1 + \Delta t\gamma A1\left(u^j\right) + \Delta t\gamma A2\left(u^j\right)\right] f^{j+1/2} + \\ + \left[\Delta t\gamma A3\left(f^j\right) + \Delta t\gamma A4\left(f^j\right)\right] u^{j+1/2} = \\ = -AP1\left(u^j, f^j\right) - AP2\left(f^j, u^j\right)\left[\frac{1}{B}\Delta t\beta C + \frac{\alpha-1}{g}D2(w^j)\right] f^{j+1/2} + \\ + \frac{1}{g}B2 + \frac{\alpha}{g}\Delta t\beta\left(A5\left(u^j\right) + A6\left(u^j\right)\right) + \\ + \frac{1}{C^2R}2\Delta t\beta D1(u^j)u^{j+1/2} = \\ = L_{j+1/2} - \frac{\alpha}{g}AP3(u^j, u^j) - \frac{1}{B}CP(f^j) - \frac{1}{C^2R}DP(u^j, u^j) \\ u^{j+1} = u^j + \Delta t u^{j+1/2}, f^{j+1} = f^j + \Delta t f^{j+1/2}. \end{cases} \tag{21}$$

In this system, the values of the parameters of recurrent equations γ and β we choose from the conditions of their stability and provide the desired accuracy.

6. Equation of motion of water in the channel in the approach of the kinematic wave

So, the simplified equations of water in the form of equations of the kinematic wave [3]

$$\frac{\partial F}{\partial t} + \frac{3}{2}C\sqrt{UF}\frac{\partial F}{\partial x} - \frac{1}{Re}\frac{\partial^2 F}{\partial x^2} = Bw, \tag{22}$$

where $F = F(x,t)$ – cross-sectional area; $B = b_+(x,y) - b_-(x,y) = const$ – width of the channel; w – side inflow; Re – Reynolds number; i – slope of the bottom.

Initial and boundary conditions:

$$\begin{aligned} &F_{t=0} = F_0, \\ &\left(-\beta\tfrac{\partial F}{\partial x} + (1-\beta)\,F\right)\Big|_{x=0} = 0, \\ &\left(\gamma\tfrac{\partial F}{\partial x} + (1-\gamma)\,F\right)\Big|_{x=L} = 0, \quad \gamma, \beta > 0. \end{aligned} \tag{23}$$

Enter the denotation

$$\begin{aligned} &(h,\varphi) := \int\limits_{\Omega} h\varphi dx, \quad c\,(h,\varphi) := \int\limits_{\Omega} \nabla h \cdot \nabla\varphi dx, \\ &b\,(\xi; h, \varphi) := \int\limits_{\Omega} \xi^{m-1} h\alpha \cdot \nabla\varphi dx, \end{aligned} \tag{24}$$

$$\langle l, \varphi \rangle := \int\limits_{\Omega} R\varphi dx - \int\limits_{p} \hat{q}\varphi dx, \;\; \forall \xi, h, \varphi \in V \tag{25}$$

Taking into account the designation (24), (25), variational formulation of the problem will look like:

$$\begin{cases} Given: \;\; h^0 \in V \; and \; \lambda \in (0,1]; \\ Find: \;\; H^{k+\frac{1}{2}} \in V, \\ (H^{k+\frac{1}{2}}, \varphi) + \Delta t \lambda (mb(h^k; H^{k+\frac{1}{2}}, \varphi) + \frac{1}{Re} c(H^{k+\frac{1}{2}}, \varphi)) = \\ = \left\langle l_{k+1/2}, \varphi \right\rangle - b(h^k; h^k, \varphi) - \frac{1}{Re} c(h^k, \varphi) \quad \forall \varphi \in V, \\ h^{k+1} = h^k + \Delta t H^{k+\frac{1}{2}}, \;\; k = 0, ..., N_T. \end{cases} \tag{26}$$

The constructed variational problem of channel flow in kinematic approximation (26) makes it possible to find the depth of the flow in any point of time.

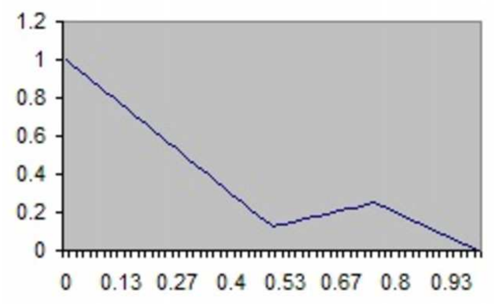

FIG. 3. Form of the bottom of the channel

7. Analysis of numerical experiments

We will test the obtained models on the test examples. The first example shows an effective use of quadratic approximations to eliminate the oscillation of solutions of hydrodynamic problem. The second example shows finding a solution of the problem of kinematic approximation, taking into account the

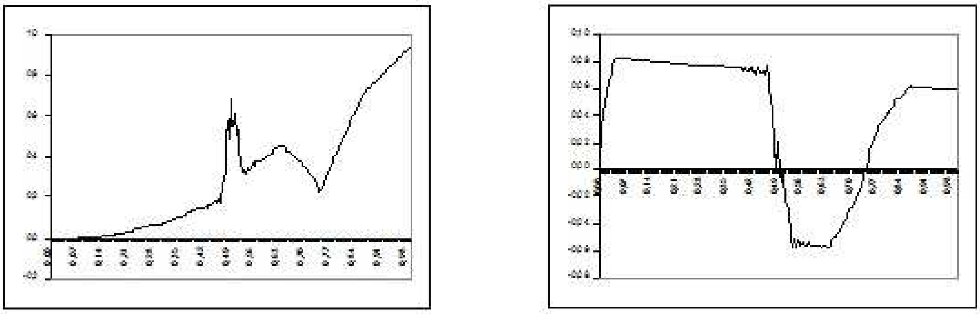

FIG. 4. Cross-sectional area and velocity (linear approximation 1000FE)

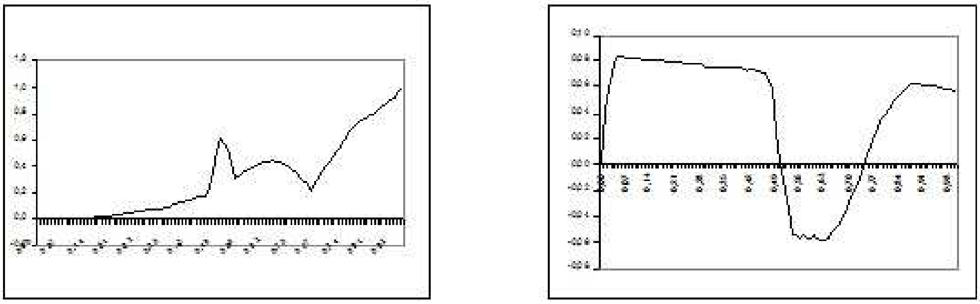

FIG. 5. Cross-sectional area and velocity (quadratic approximation 500FE)

addition with the second derivative. But the line of the middle bottom in examples 1 and 2 is the same.

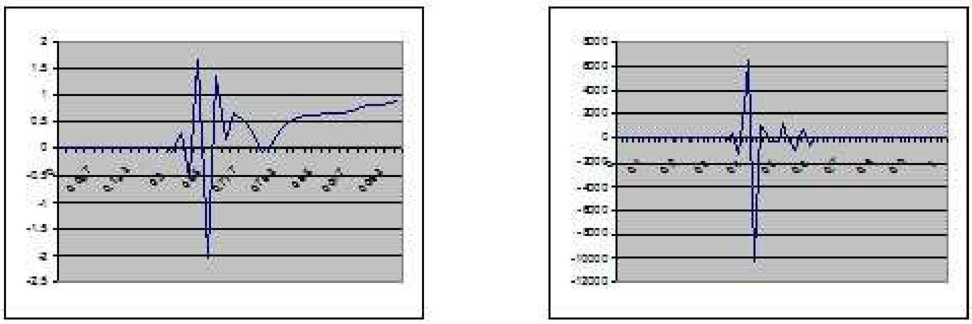

FIG. 6. Cross-sectional area and velocity (kinematic wave $\frac{1}{Re} = 0$)

Example 1. Input data: α=1, $0 \leq x \leq 1$, $0 \leq t \leq 1$, $\Delta t = 0.0001$, B=8, g=9.8, C=60, R=1, $F_0 = x^2$.

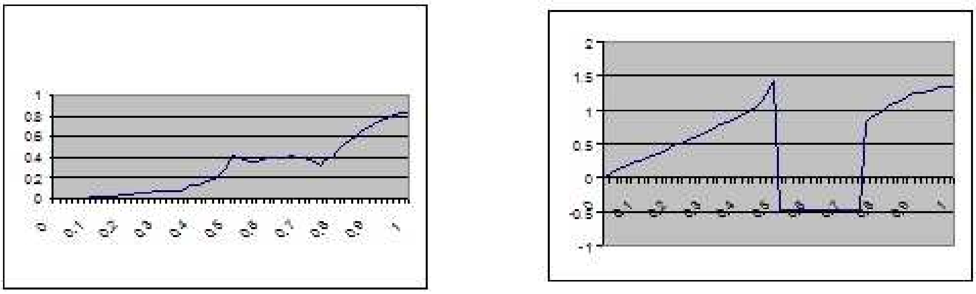

FIG. 7. Cross-sectional area and velocity (kinematic wave $Re = 20$)

Example 2. Input data: α=1, $0 \le x \le 1$, $0 \le t \le 1$, $\Delta t = 0.0001$, B=20, g=9.8, C=60, R=0, 1/Re=0 and Re = 20, $F(0,t) = 0$, $\frac{\partial F}{\partial x}\big|_{x=1} = 0$, $F_0 = x^2$, $U = C\sqrt{VF}$.

8. CONCLUSIONS

In this paper, a model of fluid motion in open pseudo prismatic channel in the hydrodynamic approximation, which is described by a system of equations with unknown variables of velocity and area cross-section of the flow, was constructed. In conditions of balance of the forces of resistance and gravity for this model the equation of the kinematic wave was written. In it the addition with the number of Reynolds and with the second derivative for spatial variable was taking into account. The initial-boundary problem was set for both approaches and its variational formulation was written. The variational problem was solving using the finite elements method. The choice of linear and quadratic basis functions was investigated in discretization a problem for a spatial variable and in application one-time recurrent integration scheme in time.

The obtained solutions of the problem are tested on examples with a complex relief of the bottom of the channel. In the model of hydrodynamic approximation, the expediency of increasing the order of approximation schemes for a spatial variable in approximations of velocity of flow and in the kinematic wave model the use of a regularized multiplier are shown. The test example shows a comparison of the two approaches, taking into account the change of the gradients of the line of the middle bottom of the channel.

BIBLIOGRAPHY

1. Burakov D. A. Two-dimensional nonstationary mathematical model of water catchment / D. A. Burakov, E. D. Karepova, V. V. Shaidurov // Notes on Numerical Fluid Mechanics and Multidisciplinary Design. – 2011. – № 115. – P. 253-266.
2. Venherskyi P. S. One of approaches of modeling of processes of river-bed flow of fluid / P. S. Venherskyi, Y. V. Kokovska // Visnyk of Lviv University. – Ser. of App. Math. and Inform. – 2009. – № 15. – P. 178-195. (in Ukrainian).
3. Venherskyi P. S. The equation of the surface flow in the kinematic waves approximation for channels with uneven bottom / P. S. Venherskyi, V. M. Kyrylych, Y. V. Kokovska // Applied problems of mechanics and mathematics. – 2016. – № 14. – P. 31-36.
4. Hladkyi A. V. Numerical-analytical methods for wave processes studies. / A. V. Hladkyi, I. V. Sergijenko, V. V. Skopeckyi. – Science opinion. – Kyiv. – 2001. – 452 p.

5. Kartvelishvili N. A. Flows in undeformed channels / N. A. Kartvelishvili. – L.: Gidrometeoizdat. – 1973. – 279 p.
6. Kokovska Y. V. On comparison of two approaches of numerical modelling of channel flow of fluid / Y. V. Kokovska, P. S. Venherskyi // XXI All-Ukrainian Scientific Conference "Modern Problems of Applied Mathematics and Informatics". – Collection of scientific works. – Lviv. – 2015. – P. 183-184.
7. Kokovska Y. Numerical solution of variational problems of surface and channel flows by finite elements method / Y. Kokovska, M. Prytula, P. Venherskyi // Ukrainian International Conference on Applied Mathematics. – Lviv. – 2017. – P. 86-87.
8. Kokovska Y. Use linear and quadratic approximations for solution of problem of fluid flow in chanel / Y. Kokovska // Visnyk of Lviv. University. – Ser. of App. Math. and Inform. – 2012. – No 18. – P. 165-175. (in Ukrainian).
9. Kokovska Y. Mathematical modeling of fluid flows in pseudoprismatic channels. / Y. Kokovska // Modern Problems of Mechanics and Mathematics: Ya.S.Pidstrygach Institute of Applied Problems of Mechanics and Mathematics of the National Academy of Sciences of Ukraine. – Lviv. – 2011. – P. 189-190.
10. Kuchment L. S. Dynamic-stochastic models of river flow formation / L. S. Kuchment, A. N. Gelfan. – RAN, Institute of Water Problems. – M.: Science. – 1993. – 112 p.
11. Ladikov-Royev Yu. P. Mathematical modeling of the process of formation of stream flow and stream erosion of soils / Yu. P. Ladikov-Royev, V. I. Panchuk, G. A. Chechko // Management and Informatics Problems. – 2000. – № 3. – P. 76-85.
12. Savula Y. G. Numerical analysis of the problems of mathematical physics by variational methods / Y. G. Savula // I. Franko National univ. of Lviv. – Lviv. – 2004. – P. 222.
13. Susidko M. M. Mathematical modeling of formation processes as a basis for forecasting systems / M. M. Susidko // Science Collection of the KNU "Hydrology, Hydrochemistry and Hydroecology". – 2000. – Vol. 1. – P. 32-40.
14. Shynkarenko G. A. Projection-net methods for solving initial-boundary problems / G. A. Shynkarenko. – Kiev: NMC VO. – 1991. – 87 p.

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